Daily Sequences Drill #7

Sheet by David Akinyele-Aje

Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Mixed Synthesis Capstone · Every technique from Days 1–6, combined.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

One quick question per technique from the whole week. Pure recognition, full speed.

1. Write down the nth term of the AP $4, 9, 14, 19, \ldots$

2. Evaluate $\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$.

3. A GP has $a = 6, r = \frac{1}{3}$. Find its sum to infinity.

4. Write down the characteristic equation of $a_n = 7a_{n-1} - 10a_{n-2}$.

5. Find the fixed point of $f(x) = 3x - 8$.

6. If the mean of the first 5 terms of a sequence is an integer, what can be said about their sum?

7. Is $2, 6, 18, 54, \ldots$ an AP or a GP?

8. A recurrence has both characteristic roots with magnitude < 1. What happens to the sequence as $n \to \infty$?

9. Find the coefficient of x^2 in $(1 + x)^5$.

10. Write down the fixed-point formula for $f(x) = rx - c$ $(r \neq 1)$.

Section B Manipulation Drills (≤50 s each)

Each question pairs two (or more) techniques from different days.

1. An AP has first term 2, common difference 3. Verify that it also satisfies the recurrence $a_n = 2a_{n-1} - a_{n-2}$.

2. Using $(1 - x)^{-1} = \sum x^k$ and $x = \frac{1}{4}$, evaluate $\displaystyle\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k$.

A) $\frac{4}{3}$

B) $\frac{3}{4}$

C) 4

D) $\frac{1}{4}$

3. A recurrence has characteristic roots $\frac{1}{2}$ and $\frac{1}{3}$, with no forcing term. Does the sequence converge, and to what?

A) Converges to 0.

B) Converges to $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

C) Diverges.

D) Oscillates without converging.

4. A sequence satisfies $a_{n+1} = \dfrac{1}{2 - a_n}$. Find its fixed point(s), and determine whether $a_1 = 1$ leads to a constant sequence.

5. The sum of a binomial row, $\sum_k \binom{n}{k}$, and the sum of an AP's terms, $\sum_k a_k$, are both examples of what general mathematical operation?

A) Summing a finite sequence.

B) They are unrelated concepts.

C) Both are geometric series.

D) Both require calculus.

6. In the alternating multiply-subtract process (Day 6), the shift $b_n = a_n - \frac{c}{r-1}$ turns Alice's map into exactly which type of sequence from Day 3?

A) A GP with ratio r.

B) An AP with difference r.

C) A GP with ratio c.

D) Neither.

7. The series $(1 - x)^{-1} = \sum x^k$ is the closed form of which simple recurrence? State the recurrence and its characteristic root.

8. Which of Day 1's arithmetic sequences is also strictly monotonic (Day 5's sense) for every valid common difference $d \neq 0$?

A) Every AP with $d \neq 0$ is monotonic.

B) Only APs with $d > 0$.

C) No AP is ever monotonic.

D) Only APs with integer d.

9. A GP has $|r| > 1$. Is the sequence bounded?

A) No — it diverges in magnitude, unbounded.

B) Yes, always bounded.

C) Bounded only if $a > 0$.

D) Bounded only if r is an integer.

10. Day 6's "$a_k = 1$ for all k" construction and Day 2's binomial row-sum fact $\sum_k \binom{n}{k} = 2^n$ are both about sums. Are these two facts related, or independent observations?

Section C Substitution & Structure (≤90 s each)

Tricky synthesis MCQs, combining techniques from across the week.

1. An AP and a GP both have first term 3; the AP has $d = 4$, the GP has $r = 2$. After how many terms (if ever) could they next coincide?

A) They never coincide again.

B) At term 4.

C) At term 6.

D) At term 10.

2. Compare the growth of $\sum_{k=1}^{n} a_k$ where $a_k = 1$ for all k (Day 6's construction) against $\sum_{k=0}^{n} \binom{n}{k}$ (Day 2's row sum) as $n \to \infty$. Which grows faster?

A) The binomial row sum grows faster (exponentially vs. linearly).

B) They grow at the same rate.

C) The Day 6 sum grows faster.

D) Neither grows — both are constant.

3. A recurrence $a_n = pa_{n-1} + qa_{n-2}$ has both characteristic roots equal to a Day 6-style fixed point $\frac{c}{r-1}$. If $\frac{c}{r-1} = 1$, what does Day 4's result about repeated roots equal to 1 tell us about the long-term behaviour of (a_n)?

A) Linear growth in n (effectively an AP).

B) Exponential growth.

C) Convergence to 0.

D) Periodic oscillation.

4. Evaluate $\displaystyle\sum_{k=0}^{\infty}(k+1)\left(\frac{1}{3}\right)^k$ using Day 2's identity $(1-x)^{-2} = \sum_{k=0}^{\infty}(k+1)x^k$.

A) $\frac{9}{4}$

B) $\frac{3}{2}$

C) $\frac{4}{9}$

D) $\frac{2}{3}$

5. A Möbius map (Day 5) is used as the combined operation in an alternating writing process (Day 6). If the map has order exactly 3 under iteration, what is guaranteed about EVERY starting value (where the map is defined)?

A) Every starting value returns to itself after exactly 3 applications.

B) Only some starting values return; others diverge.

C) The map converges to a fixed point instead.

D) Nothing general can be said.

6. For a GP with ratio r, for which values of r is the sequence BOTH monotonic AND convergent?

A) $0 \leq r \leq 1$.

B) $-1 < r < 1$.

C) Only $r = 1$.

D) $r > 1$ only.

7. Day 6's two-branch search ($u(r^k - 1) = -c$) and Day 4's root-magnitude convergence condition are both examples of what general principle about a term r^k (or r^n)?

A) Whether $|r| < 1$, $= 1$, or > 1 entirely determines the long-term size of r^k — shrinking to 0, staying constant, or blowing up.

B) Exponential terms are always eventually periodic.

C) Exponential terms never affect convergence.

D) There is no shared principle.

8. Which set of results from this week are all secretly governed by the SAME underlying idea — long-term behaviour controlled by whether a key magnitude is < 1, $= 1$, or > 1?

A) Geometric series convergence (Day 3); recurrence convergence via characteristic roots (Day 4); GP boundedness (today's B9).

B) Arithmetic sequences, binomial coefficients, integer means.

C) All seven days equally.

D) None of the sheet's content shares a common idea.

Section D Challenge (SMC / BMO1 difficulty)

Full capstone proofs, drawing on every technique from the week.

1. For each positive integer $n \geq 4$, define an n-necklace to be a circular arrangement of n (not necessarily distinct) positive integers such that the product of every 4 neighbouring integers equals n. Determine the number of integers n in the range $4 \leq n \leq 1000$ for which it is possible to form an n-necklace. *(after BMO1 2019 Q2)*

2. Prove that for any starting value a_1 (a positive integer), there exists a strictly increasing sequence of positive integers $a_1 < a_2 < \cdots < a_n$, of any desired length n, such that every partial sum S_k is divisible by k.

3. In Day 6's alternating recurrence, suppose the subtracted term is not a fixed constant c but grows every turn, $c_n = cn$. Does the shift-to-fixed-point trick (a single constant $L = \frac{c}{r-1}$) still work directly?

A) No — a single fixed constant L cannot absorb a forcing term that changes every step; a particular-solution technique (Day 4's C5) matching the forcing term's form would be needed instead.

B) Yes, exactly the same L still works.

C) The process becomes undefined.

D) It only works if $r = 1$.

4. A sequence is defined by $a_1 = 1$ and, for $n \geq 1$, a_{n+1} is the SMALLEST positive integer greater than a_n such that S_{n+1} is divisible by $n + 1$. Find $a_1, \ldots, a_5$, and conjecture (with justification) a general closed form for a_n.

5. Looking back across all seven days: which day's toolkit would be LEAST directly useful for tackling D1's n-necklace problem?

A) Day 2 (Binomial & Trinomial Expansion) — D1's argument uses periodicity and perfect-power number theory, not binomial coefficients.

B) Day 5 (Behaviour of Sequences).

C) Day 1 (Arithmetic Sequences).

D) All seven days are equally essential to D1.

"Seven days of toolkits, and the only new skill today is deciding which one the question wanted."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths