Daily Sequences Drill #5

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Monotonicity & Boundedness · Fixed Points · Non-Modular Periodicity · Floor/Ceiling Sequences.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Behaviour, not formulas: is it going up, down, or nowhere?

1. Is the sequence $a_n = 5 - n$ increasing, decreasing, or neither?

2. Is the sequence $a_n = n^2$ bounded above? Bounded below?

3. Find the fixed point of $f(x) = 2x - 3$ (i.e. solve $f(x) = x$).

4. A sequence satisfies $a_{n+1} = \dfrac{1}{a_n}$. If $a_1 = 2$, find a_2, a_3, a_4.

5. Compute $\lfloor 3.7 \rfloor$ and $\lceil 3.2 \rceil$.

6. Recall from Day 4 that a recurrence with both characteristic roots inside the unit circle converges to 0. Does $a_n = \left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n$ converge as $n \to \infty$? To what value?

7. Is the sequence $a_n = (-1)^n$ bounded? Does it converge?

8. True or false: every bounded sequence converges.

9. Is the sequence $a_n = \dfrac{n}{n+1}$ increasing or decreasing? (Check a_1, a_2, a_3.)

10. Find the smallest n for which $\lfloor \sqrt{n} \rfloor = 4$.

Section B Manipulation Drills (≤50 s each)

Prove monotonicity properly; find fixed points; handle floors carefully.

1. Prove that $a_n = \dfrac{n}{n+1}$ is increasing for all $n \geq 1$ by showing $a_{n+1} - a_n > 0$.

2. Is $a_n = \dfrac{n}{n+1}$ bounded above? If so, by what value (the supremum)?

A) Yes, supremum 1.

B) Yes, supremum 2.

C) No, unbounded.

D) Yes, supremum $\frac{1}{2}$.

3. A sequence satisfies $a_{n+1} = \dfrac{1}{a_n}$ with $a_1 = 3$. Find a_{100}.

A) 3

B) $\frac{1}{3}$

C) 100

D) $\frac{1}{100}$

4. Find the fixed point(s) of $f(x) = \dfrac{1}{x}$ (i.e. solve $f(x) = x$).

5. A sequence satisfies $a_{n+1} = f(a_n)$ where f has a fixed point at $x = 5$. If $a_1 = 5$, what is a_n for every n?

A) $a_n = 5$ for all n.

B) $a_n = 5n$.

C) $a_n \to 5$ but never equals it.

D) Cannot be determined.

6. Which of the following sequences is monotonic (always increasing or always decreasing)?

A) $a_n = (-1)^n$

B) $a_n = \sin(n)$

C) $a_n = \dfrac{1}{n}$

D) $a_n = n \bmod 3$

7. Evaluate $\lfloor -2.3 \rfloor$.

A) -2

B) -3

C) 2

D) 3

8. A sequence is defined by $a_n = \left\lfloor \dfrac{n}{2} \right\rfloor$. Find a_1, a_2, a_3, a_4, a_5.

A) $0, 1, 1, 2, 2$

B) $1, 1, 2, 2, 3$

C) $0, 0, 1, 1, 2$

D) $0, 1, 2, 2, 3$

9. Determine whether $a_n = \dfrac{(-1)^n}{n}$ converges, and if so, to what limit.

10. For an AP with common difference d, under what condition on d is the sequence strictly increasing?

A) $d > 0$

B) $d < 0$

C) $d \neq 0$

D) $d \geq 0$

Section C Substitution & Structure ($\leq$90 s each)

Iterate carefully — some maps cycle exactly, some sequences hide a red herring.

1. A sequence satisfies $a_{n+1} = \dfrac{a_n - 1}{a_n + 1}$, with $a_1 = 2$. Find a_5.

A) 2

B) $\frac{1}{3}$

C) $-\frac{1}{2}$

D) -3

2. Continuing C1's sequence, what is a_{101}?

A) 2

B) $\frac{1}{3}$

C) $-\frac{1}{2}$

D) -3

3. A sequence satisfies $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$. Which is true about (a_n)?

A) Increasing and bounded above by 2; converges to 2.

B) Decreasing and bounded below by 1.

C) Not monotonic; oscillates.

D) Unbounded, diverges to infinity.

4. The first seven terms of a sequence of positive integers are: $w_1 = 10, w_2 = 15, w_3 = 18, w_4 = 22, w_5 = 31, w_6 = 36, w_7 = 41$. Consider the statement $(*)$: "If n is prime, then w_n is a multiple of 3 or a multiple of 5." What is the smallest value of n that provides a counterexample to $(*)$? *(after TMUA 2021 Paper 2 Q10)*

A) 2

B) 3

C) 5

D) 7

5. A sequence is defined by $a_n = \lfloor \sqrt{n} \rfloor$. For how many values of n in the range $1 \le n \le 100$ does $a_n = 7$?

A) 13

B) 14

C) 15

D) 16

6. A sequence is defined by $a_n = n - \lfloor n/3 \rfloor \cdot 3$ (the remainder when n is divided by 3, written using floor notation). Is (a_n) periodic? If so, what is its period?

A) Periodic, period 3.

B) Periodic, period 2.

C) Not periodic.

D) Periodic, period 1 (constant).

7. A sequence satisfies $a_{n+1} = a_n^2 - a_n + 1$ for $n \ge 1$, with $0 < a_1 < 1$. Is (a_n) increasing, decreasing, or could it be either depending on a_1?

A) Always increasing (or constant), for any starting value.

B) Always decreasing.

C) Depends on a_1.

D) Cannot be determined without more information.

8. For a sequence defined by $a_{n+1} = f(a_n)$ with f continuous, if (a_n) converges to a limit L, what must be true about L?

A) $f(L) = L$ — L must be a fixed point of f.

B) $L = a_1$ always.

C) $f(L) = 0$.

D) No general statement can be made.

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant route — find it.

1. A sequence satisfies $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$ (as in C3). Prove rigorously, by induction, that (a_n) is bounded above by 2 for all n; then prove (a_n) is strictly increasing; and hence explain why the sequence must converge, identifying its limit.

2. A Möbius map $f(x) = \dfrac{1}{1-x}$ generates a sequence via $a_{n+1} = f(a_n)$. Starting from $a_1 = 2$, find the period of the resulting sequence (the smallest $k > 0$ such that $a_{n+k} = a_n$ for all n).

A) 2

B) 3

C) 4

D) 6

3. A sequence is defined by $a_1 = 100$ and $a_{n+1} = \lfloor a_n/2 \rfloor$. Find the smallest n for which $a_n = 0$.

A) 6

B) 7

C) 8

D) 9

4. A sequence satisfies $a_1 = 3$ and $a_{n+1} = \dfrac{a_n + \frac{4}{a_n}}{2}$. Which best describes its behaviour?

A) Decreasing (after the first term) and bounded below by 2; converges rapidly to 2.

B) Increasing, converges to 2.

C) Oscillates between values above and below 2.

D) Diverges to infinity.

5. A sequence satisfies $a_{n+1} = a_n(1 - a_n)$ with $a_1 = \frac{1}{2}$. Compute a_2, a_3, and determine whether the sequence is monotonic.

A) Decreasing for all n, since $a_{n+1} - a_n = -a_n^2 \leq 0$ always.

B) Increasing for all n.

C) Not monotonic.

D) Constant.

"A sequence that is bounded and monotonic converges. A student who is bounded and monotonic does not."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths