Daily Sequences Drill #4

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Recurrence Relations · The Characteristic Equation · Closed-Form Solutions.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Recurrences, characteristic equations, and closed-form shapes on sight.

1. A recurrence has $a_1 = 2$ and $a_{n+1} = 3a_n - 1$. Find a_2 and a_3.

2. A recurrence has $a_1 = 1$ and $a_{n+1} = 2a_n + 1$. Find a_4.

3. Write down the characteristic equation of $a_n = 5a_{n-1} - 6a_{n-2}$.

4. Solve $x^2 - 5x + 6 = 0$ for its roots.

5. A recurrence has characteristic roots 2 and 3. Write down the general form of its closed-form solution.

6. The recurrence $a_{n+1} = \frac{1}{2}a_n$, $a_1 = 8$ is secretly which type of sequence from an earlier day? Write its closed form.

7. Find the fixed point L of the recurrence $a_{n+1} = 3a_n + 4$ (i.e. solve $L = 3L + 4$).

8. True or false: every recurrence $a_n = pa_{n-1} + qa_{n-2}$ with real, distinct characteristic roots has a closed form that's a combination of two exponential terms.

9. The sequence $a_n = 2^n + 3^n$ satisfies a recurrence $a_n = pa_{n-1} + qa_{n-2}$. Find p and q.

10. A recurrence has a repeated characteristic root $r = 2$. Write down the general form of its closed-form solution.

Section B Manipulation Drills (≤50 s each)

Solve for the constants in a closed form; recognise which toolkit actually applies.

1. Solve the recurrence $a_n = 5a_{n-1} - 6a_{n-2}$ with $a_1 = 1, a_2 = 4$, for the constants A, B in $a_n = A \cdot 2^n + B \cdot 3^n$.

2. For $a_{n+1} = 3a_n + 4$, the fixed point is $L = -2$. Letting $b_n = a_n - L = a_n + 2$, what type of sequence is (b_n)?

A) A GP with ratio 3.

B) An AP with common difference 3.

C) A GP with ratio -2.

D) Neither.

3. Continuing from B2: given $a_1 = 1$ (so $b_1 = 3$), find a_5.

A) 241

B) 243

C) 239

D) 245

4. Find the characteristic equation and its roots for $a_n = a_{n-1} + 2a_{n-2}$.

5. A recurrence has characteristic roots 2 and -1, with $a_1 = 1, a_2 = 5$. Which closed form fits?

A) $a_n = 2^n + (-1)^n$

B) $a_n = 2^n - (-1)^n$

C) $a_n = 2 \cdot 2^n - (-1)^n$

D) $a_n = 2^{n-1} + (-1)^{n-1}$

6. Which of the following recurrences does NOT have a characteristic equation with two distinct real roots?

A) $a_n = 5a_{n-1} - 6a_{n-2}$

B) $a_n = 4a_{n-1} - 4a_{n-2}$

C) $a_n = a_{n-1} + 2a_{n-2}$

D) $a_n = 3a_{n-1} - 2a_{n-2}$

7. A recurrence has repeated characteristic root $r = 3$, with $a_1 = 6, a_2 = 27$. Using $a_n = (A+Bn)3^n$, find A and B.

A) $A = 1, B = 1$

B) $A = 2, B = 1$

C) $A = 1, B = 2$

D) $A = 0, B = 2$

8. Using the closed form from B7, find a_4.

9. A recurrence is defined by $a_{n+1} = a_n + 5$, $a_1 = 2$. Is this recurrence's closed form better found using the AP toolkit (Day 1) or the characteristic-equation toolkit (Day 4)?

A) The AP toolkit — it's simply $a_n = 2 + 5(n - 1)$.

B) The characteristic-equation toolkit — solve $x = x + 5$.

C) Both give the same closed form, but the characteristic equation is faster.

D) Neither toolkit applies.

10. A recurrence has characteristic equation $x^2 - 6x + 9 = 0$. What is special about its roots?

A) Two distinct real roots.

B) A repeated real root.

C) Two complex roots.

D) No real roots.

Section C Substitution & Structure (≤90 s each)

Recognise the right technique before grinding through algebra.

1. Which substitution transforms $a_{n+1} = 4a_n - 3$ into a pure geometric recurrence $b_{n+1} = 4b_n$?

A) $b_n = a_n - 1$

B) $b_n = a_n + 1$

C) $b_n = a_n - 3$

D) $b_n = a_n/4$

2. The recurrence $a_{n+1} = 4a_n - 3$ with $a_1 = 2$ has closed form $a_n = 1 + c \cdot 4^{n-1}$. Find c.

A) 1

B) 2

C) 3

D) 4

3. A recurrence has characteristic roots $r_1 = 5$ and $r_2 = -2$. Using $p = r_1 + r_2$ and $q = -r_1 r_2$, write down the recurrence $a_n = pa_{n-1} + qa_{n-2}$.

A) $a_n = 3a_{n-1} + 10a_{n-2}$

B) $a_n = 3a_{n-1} - 10a_{n-2}$

C) $a_n = -3a_{n-1} + 10a_{n-2}$

D) $a_n = 7a_{n-1} - 10a_{n-2}$

4. A recurrence has characteristic equation $x^2 - 2x + 5 = 0$. What can be said about its roots?

A) Two distinct real roots.

B) A repeated real root.

C) Two complex conjugate roots, $1 \pm 2i$.

D) No roots exist.

5. Which technique is most appropriate for solving $a_{n+1} = 2a_n + 3^n$ (note the non-constant additive term)?

A) The constant-shift trick ($b_n = a_n - L$ for a fixed L) — works directly.

B) The homogeneous characteristic-equation method, plus a particular solution of the form $C \cdot 3^n$ added to the general solution.

C) Treating it as a plain AP.

D) It cannot be solved by any method covered so far.

6. Which of the following recurrences is secretly equivalent to an arithmetic sequence — Day 1's toolkit is sufficient, no characteristic equation needed?

A) $a_{n+1} = a_n + 7$

B) $a_{n+1} = 2a_n$

C) $a_{n+1} = 2a_n + 7$

D) $a_n = 5a_{n-1} - 6a_{n-2}$

7. A recurrence $a_n = pa_{n-1} + qa_{n-2}$ has BOTH characteristic roots equal to 1. What does this imply about a_n as $n \to \infty$ (assuming (a_n) is not identically zero)?

A) Exponential growth.

B) Linear growth in n (or constant) — effectively an AP.

C) The sequence must be periodic.

D) The sequence must converge to a finite limit.

8. A recurrence has characteristic roots r and $\frac{1}{r}$ (reciprocal roots, $r \neq \pm 1$). What can be said about p and q in $a_n = pa_{n-1} + qa_{n-2}$?

A) $q = -1$ always, while $p = r + \frac{1}{r}$ can be any value with $|p| \geq 2$.

B) $p = -1$ always.

C) p and q must both equal 1.

D) No general statement can be made.

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant route — find it.

1. A sequence is defined by $a_1 = 1, a_2 = 2$, and for $n \geq 3$: $a_n = \dfrac{a_{n-1} + a_{n-2}}{2}$. Find $\lim\limits_{n\to\infty} a_n$.

2. A recurrence satisfies $a_{n+1} = 6a_n - 9a_{n-1}$ (for $n \geq 2$), with $a_1 = 1, a_2 = 6$. Using the repeated-root closed form $a_n = (A + Bn)3^n$, find a_5.

A) 405

B) 243

C) 324

D) 486

3. A recurrence has characteristic equation $x^2 + 4 = 0$ (i.e. $a_n = -4a_{n-2}$). What is true about its behaviour?

A) The sequence oscillates in sign while its magnitude grows like 2^n — consistent with complex roots of magnitude 2.

B) The sequence converges to 0.

C) The sequence is eventually constant.

D) The sequence is purely real-valued growth with no oscillation.

4. For which of the following (p, q) pairs does $a_n = pa_{n-1} + qa_{n-2}$ produce a closed form that reduces exactly to Day 1's arithmetic-sequence form (i.e. a_n linear in n), for suitable initial conditions?

A) $(p, q) = (2, -1)$

B) $(p, q) = (1, 1)$

C) $(p, q) = (-2, 1)$

D) $(p, q) = (3, -2)$

5. A recurrence has characteristic roots r_1, r_2 with $|r_1| < 1$ and $|r_2| < 1$. What happens to a_n as $n \to \infty$, regardless of initial conditions?

A) $a_n \to 0$ always.

B) a_n diverges to infinity.

C) a_n oscillates forever without settling.

D) It depends on the initial conditions.

"Every recurrence is a machine. The characteristic equation is the manual you skipped."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths