Daily Sequences Drill #3

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Geometric Sequences & Series · Sum to Infinity · The Convergence Condition $|r| < 1$.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Ratios, terms, and the sum-to-infinity formula on sight.

1. Write down the common ratio of the GP $3, 6, 12, 24, \ldots$

2. A GP has first term 5 and common ratio 2. Find the 6th term.

3. A GP has first term 2 and common ratio 3. Find S_5.

4. State the condition on r for an infinite GP's sum to converge.

5. A GP has first term 8 and common ratio $\frac{1}{2}$. Find its sum to infinity.

6. Which of these three sequences is geometric: (i) $2, 4, 6, 8$ (ii) $2, 4, 8, 16$ (iii) $2, 4, 7, 11$?

7. Write down the sum-to-infinity formula for a GP with first term a and ratio r (where $|r| < 1$).

8. True or false: a GP with common ratio $r = 1$ has a well-defined sum to infinity.

9. A GP has first term 100 and common ratio $-\frac{1}{2}$. Find its sum to infinity.

10. Write the next term of the GP $81, 27, 9, 3, \ldots$

Section B Manipulation Drills (≤50 s each)

Solve for the GP's parameters, then use them — always check convergence first.

1. A GP has first term 4 and common ratio 3. Find the smallest n for which $S_n > 1000$.

2. For which values of r does the infinite GP with first term 5 have a finite sum?

A) all real r

B) $r > 0$ only

C) $-1 < r < 1$

D) $r \neq 1$

3. A GP has first term 3 and common ratio $\frac{2}{3}$. Find its sum to infinity.

4. A GP has sum to infinity 20 and first term 5. Find r.

A) $\frac{3}{4}$

B) $\frac{1}{4}$

C) $\frac{4}{5}$

D) $-\frac{3}{4}$

5. A GP has first term 1 and sum to infinity 2. Find the second term.

A) $\frac{1}{2}$

B) $\frac{1}{4}$

C) 2

D) 1

6. An AP and a GP both have first term 4. The AP has common difference 2; the GP has common ratio 2. Which sequence has the larger 10th term?

A) The AP

B) The GP

C) They are equal

D) Cannot be determined

7. A GP has all terms positive and common ratio $r > 1$. Does its sum to infinity exist?

A) Yes, always.

B) No, never — it diverges.

C) Only if the first term is small.

D) Only if $r < 2$.

8. Express the recurring decimal $0.\overline{3} = 0.333\ldots$ as a fraction, using the geometric series sum-to-infinity formula.

9. A GP has first term 2 and common ratio -3. Find S_4.

A) 40

B) -40

C) 160

D) -160

10. Recall from Day 2 that $(1-x)^{-1} = 1 + x + x^2 + \cdots$ for $|x| < 1$. Using $x = \frac{1}{3}$, what does this series sum to?

A) $\frac{3}{2}$

B) $\frac{2}{3}$

C) 3

D) 2

Section C Substitution & Structure (≤90 s each)

Convergence first, computation second.

1. A GP has first term 1 and unknown common ratio r. Its sum to infinity exists and equals 4. What must be true about r?

A) $r = \frac{3}{4}$ exactly.

B) $-1 < r < 1$, but no exact value is determinable.

C) $r = \frac{4}{3}$.

D) Infinitely many values of r satisfy this.

2. A geometric sequence has first term a and common ratio r, both positive integers with $r > 1$. Let S_n denote the sum of the first n terms. Given $S_{18} - S_{12} = kS_6$ for some positive integer k, find the smallest possible value of k. *(after TMUA 2022 Paper 1 Q8)*

A) 2^6

B) 2^{12}

C) 2^{18}

D) $\dfrac{2^6}{2^6 - 1}$

E) $2^6(2^6 - 1)$

3. An infinite geometric series has first term $a = 6$ and common ratio $r = \dfrac{3k}{10}$, where k is chosen uniformly at random from the integers $-5, -4, \ldots, 4, 5$ (11 equally likely values). Find the probability that the series converges *and* its sum to infinity exceeds 5. *(after TMUA 2023 Paper 1 Q18)*

A) $\frac{4}{11}$

B) $\frac{5}{11}$

C) $\frac{7}{11}$

D) $\frac{3}{11}$

4. The series $\sum_{k=1}^{\infty} ar^{k-1}$ converges (for real a, r) when $|r| < 1$. If a and r are instead allowed to be complex numbers, which is the correct convergence condition?

A) $|r| < 1$ (the same condition, using the complex magnitude).

B) $r < 1$ (a real inequality).

C) $a < 1$.

D) There is no meaningful convergence condition for complex r.

5. Convert the recurring base-2 number $0.\overline{0111} = 0.0111\,0111\,0111\ldots$ (base 2) to a fraction in base 10. *(after TMUA 2023 Paper 2 Q15)*

A) $\frac{1}{3}$

B) $\frac{1}{5}$

C) $\frac{7}{15}$

D) $\frac{7}{16}$

6. Which of the following statements about arithmetic and geometric series is FALSE?

A) An AP's terms grow linearly in n; a GP's terms with $|r| > 1$ grow exponentially in n.

B) An AP diverges to $\pm\infty$ (or stays constant) as $n \to \infty$, unless $d = 0$.

C) A GP can converge to a finite sum to infinity even though no individual term is exactly zero, provided $|r| < 1$.

D) Every convergent GP has all its terms equal to zero eventually.

7. Using $(1-x)^{-1} = \sum_{k=0}^{\infty} x^k$ from Day 2, evaluate $\displaystyle\sum_{k=0}^{\infty} \frac{1}{5^k}$ by choosing an appropriate value of x.

A) $\frac{5}{4}$

B) $\frac{4}{5}$

C) 5

D) $\frac{1}{4}$

8. Let $S = 1+2x+3x^2+4x^3+\cdots$. Recall from Day 2 that this is exactly $(1-x)^{-2} = \sum_{k=0}^{\infty}(k+1)x^k$. Evaluate S at $x = \frac{1}{4}$.

A) $\frac{16}{9}$

B) $\frac{4}{3}$

C) $\frac{9}{16}$

D) $\frac{3}{4}$

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant route — find it.

1. Prove that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2^k} = 1$ exactly (not merely "approaches 1"), using the sum-to-infinity formula.

Then explain why the partial sum $S_n = \sum_{k=1}^{n} \dfrac{1}{2^k} = 1 - \dfrac{1}{2^n}$ never actually equals 1 for any finite n.

2. Let $x = 0.\overline{9} = 0.999\ldots$ (a recurring 9). Writing x as an infinite geometric series and summing it, determine x exactly.

A) $x = 1$ exactly.

B) x is infinitesimally less than 1 but not equal to it.

C) $0.\overline{9}$ is undefined.

D) $x = \frac{9}{10}$.

3. A ball is dropped from height h. Each time it bounces, it rebounds to a fraction r of its previous height ($0 < r < 1$). Find, in terms of h and r, the total distance the ball travels (up and down, forever) before coming to rest.

A) $h \cdot \dfrac{1+r}{1-r}$

B) $h \cdot \dfrac{2}{1-r}$

C) $h \cdot \dfrac{1}{1-r}$

D) $h(1+2r)$

4. The sum to infinity of a GP equals the sum of its first two terms. What can be said about the common ratio r?

A) $r = 0$ (the "GP" is actually constant).

B) $r = \pm 1$.

C) $r = \frac{1}{2}$.

D) No such r exists.

5. A sequence is defined by $a_1 = 3$ and $a_{n+1} = \frac{1}{2}a_n$ for $n \geq 1$. Using the sum-to-infinity formula, evaluate $\displaystyle\sum_{n=1}^{\infty} a_n$.

A) 6

B) 3

C) 4.5

D) It cannot be summed, since it's defined by a recurrence, not a formula.

"Infinity is not a number and $r = 1$ is not a ratio you may divide by. Two mistakes, one line."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths