Daily Sequences Drill #2

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Binomial & Trinomial Expansion · Pascal's Triangle as a Sequence · The Generalized Binomial Series.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Pascal's triangle and binomial coefficients on sight.

1. Write down row 5 of Pascal's triangle (the coefficients of $(x+y)^5$).

2. Evaluate $\binom{6}{2}$.

3. Write down $\displaystyle\sum_{k=0}^{n}\binom{n}{k}$ in closed form.

4. Expand $(1+x)^3$ fully.

5. Find the coefficient of x^2 in $(1+x)^6$.

6. Row 4 of Pascal's triangle is $1, 4, 6, 4, 1$. Is this sequence of five numbers arithmetic? Justify with one check.

7. Simplify $\binom{n}{0}+\binom{n}{1}$.

8. State the value of $\displaystyle\sum_{k=0}^{n}(-1)^k\binom{n}{k}$ for $n \geq 1$.

9. Find the constant term in $(1+x)^5$.

10. True or false: row n of Pascal's triangle always has exactly $n+1$ entries.

Section B Manipulation Drills (≤50 s each)

Substitute into the binomial expansion; extract specific coefficients.

1. Prove that $\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}=2^n$ by substituting a value into the expansion of $(1+x)^n$.

2. Row 10 of Pascal's triangle sums to 2^{10}. If instead you sum only its first four entries, $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3}$, is the result arithmetic, geometric, or neither, as a rule? Compute the value.

3. Using $x = -1$ in $(1+x)^n$ $(n \geq 1)$, what is $\displaystyle\sum_{k=0}^{n} (-1)^k \binom{n}{k}$?

A) 0

B) 1

C) 2^n

D) $(-1)^n$

4. Find the coefficient of x^3 in $(2+x)^5$.

A) 10

B) 40

C) 80

D) 20

5. Find the coefficient of x^4 in $(1-2x)^6$.

A) 60

B) -240

C) 240

D) -60

6. Find the constant term in the expansion of $\left(x + \dfrac{1}{x}\right)^4$.

A) 4

B) 6

C) 8

D) 1

7. In the expansion of $(1+x)^n$, the coefficient of x^2 is 21 times the coefficient of x^1. Find n.

A) 21

B) 42

C) 43

D) 44

8. The coefficients of x^2 and x^3 in the expansion of $(1+x)^n$ are equal. Find n.

A) 4

B) 5

C) 6

D) Cannot be determined.

9. Find the coefficient of x^2 in the expansion of $(1 + 2x + 3x^2)^3$.

A) 15

B) 18

C) 21

D) 24

10. The row-sums of Pascal's triangle are $1, 2, 4, 8, 16, \ldots$ Is this sequence of row-sums arithmetic, geometric, both, or neither? State the relevant ratio or difference.

Section C Substitution & Structure

($\leq$90 s each)

Structure over computation — what the binomial series is really telling you.

1. Which of the following statements about partial sums of binomial coefficients is correct?

A) $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{k-1}$ always equals 2^n, for any $k \leq n$.

B) There is no simple closed form for a general partial sum of binomial coefficients, unlike the full row sum.

C) Partial sums of binomial coefficients are always arithmetic in k.

D) Partial sums of binomial coefficients are always geometric in k.

2. Find the coefficient of x in the expression $(1 + x)^0 + (1 + x)^1 + \cdots + (1 + x)^{40}$. *(after TMUA 2019 Paper 1 Q3)*

A) 40

B) 41

C) 820

D) 1640

E) 1681

3. Find, in terms of n, the coefficient of x^2 in the expression $(1 + x)^0 + (1 + x)^1 + \cdots + (1 + x)^n$. *(after TMUA 2019 Paper 1 Q3)*

A) $\binom{n}{3}$

B) $\binom{n+1}{3}$

C) $\binom{n+1}{2}$

D) $2^n - n - 1$

4. In the expansion of $(1 + x + x^2)^4$, what is the coefficient of x^3?

A) 12

B) 16

C) 20

D) 24

5. Which of the following is the expansion of $(1-x)^{-1}$ as an infinite series in x?

A) $1 - x + x^2 - x^3 + \cdots$

B) $1 + x + x^2 + x^3 + \cdots$

C) $1 - 2x + 3x^2 - 4x^3 + \cdots$

D) It cannot be expanded as a series.

6. The generalized binomial expansion gives $(1-x)^{-2} = \sum_{k=0}^{\infty}(k+1)x^k$. Find the coefficient of x^3.

A) 3

B) 4

C) 6

D) 10

7. Which of the following is FALSE, for a positive integer n?

A) $\displaystyle\sum_{k=0}^{n}\binom{n}{k} = 2^n$

B) $\displaystyle\sum_{k=0}^{n}(-1)^k\binom{n}{k} = 0$

C) $\displaystyle\binom{n}{k} = \binom{n}{n-k}$ for all $0 \le k \le n$

D) $\displaystyle\sum_{k=0}^{n}k\binom{n}{k} = 2^n$

8. The infinite series $1 + x + x^2 + x^3 + \cdots$ equals $(1-x)^{-1}$. Using $(1-x)^{-3} = \sum_{k=0}^{\infty}\binom{k+2}{2}x^k$, find the coefficient of x^5 in $(1-x)^{-3}$.

A) $\binom{5}{2}$

B) $\binom{6}{2}$

C) $\binom{7}{2}$

D) $\binom{8}{2}$

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant route — find it.

1. Find the coefficient of x^2y^3 in the expansion of $(1+x+y^2)^5$. *(after TMUA 2020 Paper 1 Q13)*

A) 0

B) 10

C) 20

D) 30

E) 60

2. The coefficient of x^2 in $(1+x+2x^2)^3$ equals the coefficient of x^2 in $(1-ax)^3$. Find the possible value(s) of a. *(after TMUA Practice Paper 1 Q19)*

A) $a = \pm\sqrt{3}$

B) $a = \pm 3$

C) $a = \sqrt{3}$ only

D) $a = 3$ only

3. Prove that $\displaystyle\sum_{k=0}^{n} k\binom{n}{k} = n \cdot 2^{n-1}$ for every positive integer n, by differentiating the binomial expansion of $(1+x)^n$.

4. Expanding $(1+x)^n(1+x)^m$ two different ways — first by combining the factors as $(1+x)^{n+m}$, then by multiplying the two separate expansions together and comparing the coefficient of x^k on both sides — produces which identity?

A) Vandermonde's identity: $\displaystyle\sum_{i=0}^{k}\binom{n}{i}\binom{m}{k-i} = \binom{n+m}{k}$.

B) $\displaystyle\binom{n}{k} + \binom{m}{k} = \binom{n+m}{k}$.

C) $\displaystyle\binom{n}{k}\binom{m}{k} = \binom{n+m}{k}$.

D) No identity results — the two sides are unrelated.

5. The coefficients of $(1-x)^{-1}$ are all 1 (constant in k). The coefficients of $(1-x)^{-2}$, namely $(k+1)$, form an arithmetic sequence in k. What kind of sequence, as a function of k, do the coefficients of $(1-x)^{-3} = \sum_{k=0}^{\infty}\binom{k+2}{2}x^k$ form?

A) Arithmetic.

B) Geometric.

C) Quadratic in k (neither arithmetic nor geometric).

D) Constant.

"Pascal's triangle has been doing your multiplication for four hundred years. Let it."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths