Daily Sequences Drill #1 — Answers & Investigations

Sheet by David Akinyele-Aje

Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Arithmetic Sequences & Series · nth-term & Sum Formulas · S_n as a Quadratic in n.

Section A Rapid Recognition

1. Write down the nth term of the arithmetic sequence $5, 8, 11, 14, \ldots$

Answer: $3n + 2$

Method: $a = 5$, $d = 3$. $a_n = a + (n-1)d = 5 + 3(n-1) = 3n + 2$.

> **Investigate further:** Every AP's nth term is linear in n, with the coefficient of n always equal to d. So the constant term is $a - d$. **For which APs is the nth term exactly dn, with no constant term at all?** Write down the general shape of such a sequence.

2. An AP has first term 7 and common difference -3. Find the 20th term.

Answer: -50

Method: $a_{20} = 7 + 19(-3) = 7 - 57 = -50$.

> **Investigate further:** A negative d just means the "$n-1$ steps" subtract instead of add — the formula never changes. **Which is the first negative term of this sequence?** Then generalise: for an AP with $a > 0$ and $d < 0$, find a rule (using a floor function) for the index of the first negative term.

3. An AP has first term 4 and common difference 5. Find the sum of the first 10 terms.

Answer: 265

Method: $S_{10} = \frac{10}{2}(2(4) + 9(5)) = 5(8 + 45) = 5(53) = 265$.

> **Investigate further:** Cross-check with $S_n = \frac{n}{2}(a_1 + a_n)$: $a_{10} = 49$, $S_{10} = 5(4 + 49) = 265$ ✓. Now: $S_n = \frac{5n^2+3n}{2}$ for this AP. **Is S_n ever exactly 1000?** Solve the quadratic and test whether its discriminant is a perfect square — that test is the whole method.

4. A sequence has nth term $4n-1$. Write down its first three terms, and state whether the sequence is arithmetic.

Answer: 3, 7, 11; $\quad d = 4$

Method: $a_1 = 3, a_2 = 7, a_3 = 11$. Consecutive differences are both 4, so the sequence is arithmetic.

> **Investigate further:** Any nth term of the form $pn + q$ is automatically arithmetic with $d = p$. **Now suppose the nth term is $pn^2 + qn + r$. For which p, q, r is the sequence arithmetic? For which is it geometric?** The second answer is much more restrictive than the first.

5. An AP has $a_1 = 2$ and $a_5 = 18$. Find the common difference.

Answer: 4

Method: $a_5 - a_1 = 4d = 16 \Rightarrow d = 4$.

> **Investigate further:** In general $a_m - a_k = (m-k)d$ — the "steps" between two terms is the index difference, not either index. **If you are told a_1 and a_m are both integers, must d be an integer?** Find the exact divisibility condition on $m-1$ that decides it.

6. Evaluate $1 + 2 + 3 + \cdots + 50$.

Answer: 1275

Method: $S_{50} = \frac{50 \cdot 51}{2} = 1275$.

> **Investigate further:** This is Gauss's pairing trick: each of the 25 pairs sums to 51. Note $1 + 2 + \cdots + 8 = 36 = 6^2$ is a perfect square. **Find the next n after 8 for which $1 + 2 + \cdots + n$ is a perfect square.** (These are the "square triangular numbers" — they are much rarer than you would guess.)

7. An AP has $S_n = n^2 + 3n$. Find the first term a_1.

Answer: 4

Method: $a_1 = S_1 = 1^2 + 3(1) = 4$.

> **Investigate further:** More generally $a_n = S_n - S_{n-1}$ for $n \geq 2$, while $a_1 = S_1$ on its own. **Find a_n for all n here, and check it is an AP. Then: for which S_n would $a_1 = S_1$ *disagree* with the general formula?** Look hard at the constant term.

8. True or false: every arithmetic sequence with integer first term and integer common difference consists entirely of integers.

Answer: True

Method: $a_n = a_1 + (n-1)d$ is a sum of integers whenever a_1, d, n are integers, so every term is an integer.

> **Investigate further:** The converse is false: all-integer sequences need not be arithmetic (e.g. $1, 2, 4, 8, \ldots$). **Now let a_1 be an integer but $d = p/q$ a fraction in lowest terms. Which terms are integers?** Find the exact set of such n — it is a pattern, not a scattering.

9. An AP has 3rd term 13 and 9th term 37. Find the common difference.

Answer: 4

Method: 9th term − 3rd term is 6 steps of d: $37 - 13 = 24 = 6d \Rightarrow d = 4$.

> **Investigate further:** Count steps as index differences: between the 3rd and 9th term there are $9 - 3 = 6$ gaps, not 9 or 3. **Being told a_3 and a_9 pins down the AP completely. Would being told only $a_3 + a_9$ do the same?** If not, describe the whole family it leaves open.

10. An AP has first term 4 and 6th term 29. Find the common difference.

Answer: 5

Method: 6th term − 1st term is 5 steps: $29 - 4 = 25 = 5d \Rightarrow d = 5$.

> **Investigate further:** Same step-counting as A9, anchored at the first term. **Take the terms at indices $1, 6, 11, 16, \ldots$ of any AP. Do they themselves form an AP, and if so with what common difference?** Then decide whether this works for *any* arithmetic set of indices.

Section B Manipulation Drills

1. An AP has first term 3, last term 59, and 15 terms. Find the common difference and the sum of all 15 terms.

Answer: $d = 4$, $S_{15} = 465$

Method: $l = a + (n-1)d \Rightarrow 59 = 3 + 14d \Rightarrow d = 4$. $S_{15} = \frac{15}{2}(3 + 59) = \frac{15}{2}(62) = 465$.

> **Investigate further:** Given first term, last term and count, $S_n = \frac{n}{2}(a + l)$ beats finding d first. Here $S_n = 2n^2 + n$. **What is the smallest n with $S_n > 1000$?** Solve the inequality rather than tabulating — and note the answer is not what a rough $\sqrt{500}$ estimate suggests.

2. The sum of the first 8 terms of an AP is 100, and the sum of the first 4 terms is 30. What is the sum of terms 5 through 8?

A) 50

B) 70

C) 130

D) 100

Answer: B) 70

Method: $S_8 - S_4$ is exactly the sum of terms 5 through 8, by definition of S_n as a running total: $100 - 30 = 70$. No need to ever find a or d.

> **Investigate further:** The trap is solving for a, d first — a detour, since $S_8 - S_4$ is a direct subtraction. **Now find S_{12}, still without ever finding a or d.** Hint: the block sums S_4, $S_8 - S_4$, $S_{12} - S_8$ form an AP themselves. Why must they?

3. Prove that $3 + 6 + 9 + \cdots + 3n = \dfrac{3n(n+1)}{2}$ using the arithmetic series sum formula.

Answer: Proof by the AP sum formula.

Method: $3, 6, 9, \ldots, 3n$ is an AP with $a = 3$, $d = 3$, last term $3n$, and n terms. $S_n = \frac{n}{2}(a + l) = \frac{n}{2}(3 + 3n) = \frac{3n(n+1)}{2}$. $\square$

> **Investigate further:** This is $3 \times (1 + 2 + \cdots + n)$: scaling every term by k scales the sum by k. **Now use the same sum formula on the AP $1, 3, 5, 7, \ldots$ to prove that the first n odd numbers sum to exactly n^2.** It falls out in one line — no induction needed.

4. An AP has 21 terms summing to 315. Find the middle (11th) term.

A) 15

B) 315

C) 21

D) It cannot be determined without knowing a and d.

Answer: A) 15

Method: For an AP with an odd number of terms $n = 2k + 1$, the sum equals n times the middle term (the terms pair up symmetrically around it, and the middle term is left over unpaired but equal to the pair-average). Here $315 = 21 \times (\text{middle term}) \Rightarrow \text{middle term} = 15$.

> **Investigate further:** This works only because 21 is odd; an even-length AP has no single middle term. **So: a 20-term AP sums to 310. What is the average of its 10th and 11th terms?** Answer it without finding a or d, and say what the even case replaces "the middle term" with.

5. An AP has first term 10 and common difference -3. Find the last term of the sequence that is still positive.

Answer: 1

Method: $a_n = 10 - 3(n-1) > 0 \Rightarrow n < \frac{13}{3} \approx 4.33$, so the last positive term is $a_4 = 10 - 9 = 1$ (and $a_5 = -2 < 0$ confirms the sequence has already crossed zero).

> **Investigate further:** Check the boundary term itself, not just the cutoff: $n < 4.33$ gives $n = 4$, but you must confirm $a_4 > 0$ (it could have landed exactly on 0). **Keep $a_1 = 10$ but vary d. For exactly which d does this AP have precisely 4 positive terms?** You should get an interval, closed at one end and open at the other — be careful which.

6. Which of the following is NOT necessarily true for an arithmetic sequence with common difference $d > 0$?

A) The sequence is strictly increasing.

B) $a_{n+1} - a_n = d$ for every n.

C) The sequence is unbounded above.

D) All terms are positive.

Answer: D) All terms are positive.

Method: $d > 0$ guarantees the sequence is strictly increasing (A, true) and therefore unbounded above (C, true), and by definition $a_{n+1} - a_n = d$ always (B, true). But the first term a_1 can be any real number, including very negative — e.g. $a_1 = -100, d = 1$ has plenty of negative terms. So D is the one that need not hold.

> **Investigate further:** $d > 0$ constrains the *shape* of the sequence, not its starting position. **Given $d > 0$, exactly what condition on a_1 makes every term positive? And if $a_1 < 0$, how many negative terms are there?** Give the second answer as a formula in a_1 and d.

7. Two APs share the same common difference d. Their first terms differ by 6. What is the difference between their nth terms, for any n?

A) 6

B) $6 + (n-1)d$

C) $6n$

D) It depends on d.

Answer: A) 6

Method: Let the sequences be $a_n = a_1 + (n-1)d$ and $b_n = b_1 + (n-1)d$ (same d). Then $a_n - b_n = a_1 - b_1 = 6$ for every n — the $(n-1)d$ terms cancel exactly because the common difference is shared.

> **Investigate further:** With different common differences the gap grows linearly instead of staying constant — try $d_1 = 2, d_2 = 3$ to see it become $6-(n-1)$. **So if $d_1 \neq d_2$, show two APs can agree at most once, and find exactly when that crossing lands on a positive integer index.**

8. An AP has $a_3 = 11$ and $a_3 + a_5 = 32$. Find the common difference.

A) 3

B) 4

C) 5

D) 6

Answer: C) 5

Method: $a_3 + a_5 = (a+2d) + (a+4d) = 2a + 6d = 32 \Rightarrow a + 3d = 16$. Subtracting $a_3 = a + 2d = 11$: $(a+3d) - (a+2d) = 16 - 11 \Rightarrow d = 5$.

> **Investigate further:** Back-substituting gives $a = 1$; always re-check both conditions ($a_3 = 11$ ✓, $a_3 + a_5 = 32$ ✓). **Now drop the condition $a_3 = 11$ and keep only $a_3 + a_5 = 32$. Is the AP still determined?** Describe every AP satisfying it — one equation, two unknowns.

9. An AP has first term a and common difference d, both positive integers, and its 5th term equals 17. Which pair (a, d) is NOT possible?

A) $(13, 1)$

B) $(9, 2)$

C) $(5, 3)$

D) $(2, 4)$

Answer: D) $(2, 4)$

Method: Need $a + 4d = 17$ with a, d positive integers. Check each: $13 + 4(1) = 17$✓, $9 + 4(2) = 17$✓, $5 + 4(3) = 17$✓, but $2 + 4(4) = 18 \neq 17$ — D fails the defining equation outright.

> **Investigate further:** The valid pairs follow "a drops by 4 as d rises by 1": $13, 9, 5, 1$. **How many pairs (a, d) of positive integers satisfy $a + 4d = 17$ in total? Then generalise: how many satisfy $a + kd = N$?** Your general answer should be a single floor expression.

10. An AP with first term 2 and common difference 3 (terms $2, 5, 8, 11, \ldots$) and a GP with first term 2 and common ratio 2 (terms $2, 4, 8, 16, \ldots$) are compared term by term. At which term number do they next agree, within the first 6 terms of each?

A) $n = 2$

B) $n = 3$

C) $n = 4$

D) They never agree again within 6 terms.

Answer: B) $n = 3$

Method: AP: $2, 5, 8, 11, 14, 17$. GP: $2, 4, 8, 16, 32, 64$. Comparing term by term, both give 8 at the 3rd term.

> **Investigate further:** After $n = 3$ the GP overtakes permanently: geometric growth always eventually dominates linear. **Notice these two sequences also agree at $n = 1$. Why does the question ask only for the *next* agreement — and can you prove $2^n > 3n - 1$ for every $n \geq 4$?** Induction is the clean route.

Section C Substitution & Structure

1. An AP has n integer terms ($n \geq 2$) summing to 15. Which of the following must be true? *(after TMUA 2019 Paper 2 Q11)*

I. n is even.

II. The first term is odd.

III. The common difference is odd.

A) none of them

B) I only

C) II only

D) III only

E) I and II only

F) I and III only

G) II and III only

H) I, II and III

Answer: A) none of them

Method: Writing $S_n = \frac{n}{2}(2a + (n-1)d) = 15$ gives $n(2a + (n-1)d) = 30$, so n must divide 30. Counterexamples kill every statement at once: with $n = 3, a = 4, d = 1$ the sequence is $4, 5, 6$ (sum 15) — $n = 3$ is odd (kills I) and $a = 4$ is even (kills II). With $n = 3, a = 3, d = 2$ the sequence is $3, 5, 7$ (sum 15) — $d = 2$ is even (kills III). No single parity pattern is forced.

> **Investigate further:** The trap of "must be true" questions: one valid example proves nothing, but one counterexample per statement rules each out. The working also showed n must divide 30. **Use that to build an AP of exactly 6 terms summing to 15.** Negative terms are allowed — which is precisely why statements I–III all failed.

2. The sequences $a_n = 2n + 1$ and $b_n = 3n - 2$ are both arithmetic. Let $c_n = a_n + b_n$. Which is true about (c_n)?

A) It is arithmetic with common difference 5.

B) It is arithmetic with common difference 6.

C) It is not arithmetic in general.

D) It is geometric.

Answer: A) arithmetic with common difference 5

Method: $c_n = (2n+1) + (3n-2) = 5n-1$, which is linear in n with slope 5 — so (c_n) is arithmetic with $d = 5$.

> **Investigate further:** Sums of APs are always arithmetic, with d the sum of the two original common differences $(2+3=5)$. **Can the *product* of two APs ever be arithmetic? What if one of them is constant?** Multiply out the general case and look only at the n^2 coefficient — that single term settles it.

3. An AP (a_n) has common difference d. What can be said about $b_n = a_n^2$?

A) (b_n) is always arithmetic.

B) (b_n) is always geometric.

C) (b_n) is arithmetic only if $d = 0$.

D) (b_n) is arithmetic only if $a_1 = 0$.

Answer: C) arithmetic only if $d = 0$

Method: $a_n = a + (n-1)d$, so $b_n = (a + (n-1)d)^2$ is a quadratic in n with leading coefficient d^2. A quadratic sequence is arithmetic (i.e. linear in n) only if its leading coefficient vanishes: $d^2 = 0 \Rightarrow d = 0$.

> **Investigate further:** The flip side of C2: sums stay linear, but squaring introduces an n^2 term unless the AP was already constant. **Related question: for which APs is the squared sequence (a_n^2) *geometric*?** Think about what a constant ratio would force a_n/a_{n-1} to do.

4. An AP has all terms positive. The sum of the first 6 terms equals the sum of terms 7 through 10. Which must be true?

A) The common difference d must be positive.

B) d must be negative.

C) d must be zero.

D) d can have either sign.

Answer: A) d must be positive

Method: $S_6 = 6a + 15d$. Sum of terms 7–10 is $4a + (6+7+8+9)d = 4a + 30d$. Setting equal: $6a + 15d = 4a + 30d \Rightarrow 2a = 15d \Rightarrow a = 7.5d$. For $a > 0$ (all terms positive requires at least $a_1 > 0$), d must be positive too, since $a = 7.5d$ forces d and a to share a sign.

> **Investigate further:** "All terms positive" is stronger than $a > 0$ — though with $d > 0$ the rest follow once $a_1 > 0$. The condition here reduced to $2a = 15d$. **Can a and d both be integers? Find the smallest positive integer pair.** The factor of 2 against the 15 is doing all the work.

5. An AP has positive integer first term a and common difference d with $a < d$. Which pair (a, d) gives a sequence that contains the term 100 exactly?

A) $(5, 19)$

B) $(6, 17)$

C) $(7, 23)$

D) $(8, 13)$

Answer: A) $(5, 19)$

Method: Need $d \mid (100 - a)$. Check: $100 - 5 = 95 = 19 \times 5$ ✓; $100 - 6 = 94$, $94/17$ is not an integer; $100 - 7 = 93$, $93/23$ is not an integer; $100 - 8 = 92$, $92/13$ is not an integer. Only A works.

> **Investigate further:** The general rule: an AP hits target T exactly when $d \mid (T - a)$. **Option B was $(6, 17)$, which misses 100. So which term of that AP first exceeds 100, and does the AP get closer to 100 from below or from above?** Compute both neighbours before deciding.

6. An AP with first term a and common difference d satisfies $S_{2n} = 2S_n$ for some particular $n > 0$. What does this force?

A) d must equal 0.

B) a must equal 0.

C) n must be even.

D) No constraint — this holds automatically for every AP.

Answer: A) d must equal 0

Method: $S_{2n} = n(2a + (2n - 1)d)$ and $2S_n = n(2a + (n - 1)d)$. Setting them equal and dividing by $n > 0$: $2a + (2n - 1)d = 2a + (n - 1)d \Rightarrow nd = 0$. Since $n > 0$, this forces $d = 0$.

> **Investigate further:** A surprising result: doubling the term count can only double the sum if the sequence never changes. **Now show the same for tripling — and in fact prove that $S_{kn} = kS_n$ forces $d = 0$ for every integer $k \geq 2$.** The k should cancel out entirely, leaving one clean equation.

7. For an AP with common difference $d \neq 0$, the sum S_n of the first n terms, viewed as a function of n, is:

A) linear in n.

B) quadratic in n with no linear term.

C) quadratic in n.

D) exponential in n.

Answer: C) quadratic in n

Method: $S_n = \frac{n}{2}(2a + (n - 1)d) = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n$. Since $d \neq 0$, the n^2 coefficient is nonzero, so S_n is genuinely quadratic (not linear, and it does have a linear term unless $a = \frac{d}{2}$).

> **Investigate further:** The most useful fact on the sheet: a quadratic S_n with no constant term always comes from an AP, and d, a read straight off the coefficients. **Given $S_n = An^2 + Bn$, for which A, B does the AP satisfy $a_1 = d$?** Express both in terms of A and B first, then set them equal.

8. $S_n = 3n^2 - n$ is the sum of the first n terms of an arithmetic sequence. Find its common difference.

A) 3

B) 6

C) -1

D) 2

Answer: B) 6

Method: Matching $S_n = 3n^2 - n$ to $S_n = \frac{d}{2}n^2 + (a - \frac{d}{2})n$: $\frac{d}{2} = 3 \Rightarrow d = 6$, and $a - \frac{d}{2} = -1 \Rightarrow a = 2$.

> **Investigate further:** The common slips are answering 3 (forgetting the $\frac{1}{2}$) or -1 (reporting the n coefficient). **Now try $S_n = 3n^2 - n + 5$ and watch it break: compute a_1 from S_1, then a_n from $S_n - S_{n-1}$, and check whether they agree at $n = 1$.** Explain exactly which coefficient causes the failure.

Section D Challenge

1. An AP has first term a and common difference d. The sum of the first 6 terms equals the sum of the first 11 terms. Which relationship between a and d does this force? *(after TMUA 2018 Paper 1 Q2)*

A) $a = -8d$

B) $a = 8d$

C) $a = -\dfrac{17}{2}d$

D) $a = \dfrac{17}{6}d$

E) $a = -4d$

Answer: A) $a = -8d$

Method: $S_6 = S_{11} \Rightarrow \frac{6}{2}(2a+5d) = \frac{11}{2}(2a+10d) \Rightarrow 3(2a+5d) = 5.5(2a+10d) \Rightarrow 6a+15d = 11a+55d \Rightarrow -5a = 40d \Rightarrow a = -8d$.

> **Investigate further:** $S_6 = S_{11}$ says the terms from the 7th to the 11th sum to zero. **Those five terms are symmetric about one of them — so which single term must be exactly zero, and does $a = -8d$ confirm it?** Then try the real TMUA version (2018 Paper 1 Q2), which uses $S_5 = S_8$ and lands on $a = -\frac{38}{3}d$.

2. A selection S of n terms is taken from the arithmetic sequence $2, 5, 8, \ldots, 80$. What is the smallest value of n for which there must be two distinct terms in S summing to 82? *(after TMUA 2022 Paper 2 Q8)*

A) 13

B) 14

C) 15

D) 22

E) 23

F) 24

Answer: C) $n = 15$

Method: The sequence $2, 5, \ldots, 80$ has 27 terms, $a_i = 3i - 1$. Two terms with indices i, j sum to 82 exactly when $i+j = 28$. Pairing indices 1–27 by this rule gives 13 genuine pairs $(1, 27), (2, 26), \ldots, (13, 15)$, plus one unpaired index (14, whose partner would be itself). A selection can safely take one element from each of the 13 pairs plus the unpaired element — 14 terms with no pair summing to 82. Any 15th term forces a complete pair. So the smallest guaranteeing n is 15.

> **Investigate further:** Pigeonhole in disguise: the pairs are the pigeonholes, and exceeding (safe pairs + singleton) forces a full pair. Here the pairing rule was $i + j = 28$, leaving index 14 unpaired. **For which target sums does a singleton exist at all?** Answer it via the parity of $i + j$ — then check your rule against the real TMUA version (2022 Paper 2 Q8: AP $1, 4, \ldots, 70$, target 74, answer $n = 14$).

3. The first four terms of an arithmetic sequence p, $p + d$, $p + 2d$, $p + 3d$ (with $d > 0$) are all prime, and $p > 3$. Prove that d must be a multiple of 6.

Answer: Proof: see method

Method: *Mod 2:* since $p > 3$ is prime, p is odd. If d were odd, $p + d$ would be even and greater than 2, hence composite — contradiction. So d is even.
Mod 3: since $p > 3$, $p \not\equiv 0 \pmod 3$. If $d \not\equiv 0 \pmod 3$, then d is coprime to 3, so $p, p + d, p + 2d$ take all three residues mod 3 (adding a unit repeatedly cycles through every residue) — meaning one of them is $\equiv 0 \pmod 3$. That term is greater than 3 and divisible by 3, hence composite — contradiction. So $3 \mid d$.
Combining: $2 \mid d$ and $3 \mid d \Rightarrow 6 \mid d$. □

> **Investigate further:** The smallest example is $p = 5, d = 6$: $5, 11, 17, 23$, all prime. **Now show $6 \mid d$ is necessary but nowhere near sufficient — start with $p = 7, d = 6$ and see how fast it fails.** Then hunt for the next p that does work with $d = 6$.

4. An AP has $S_n = An^2 + Bn$. Given $A = 5$ and that the 10th term of the sequence is 91, find B.

A) -4

B) 4

C) -9

D) 9

Answer: A) -4

Method: $a_n = S_n - S_{n-1} = A(2n - 1) + B$ (expand and simplify $An^2 + Bn - A(n - 1)^2 - B(n - 1)$). With $A = 5$: $a_{10} = 5(19) + B = 95 + B = 91 \Rightarrow B = -4$.

> **Investigate further:** $a_n = A(2n - 1) + B$ is worth memorising: it reads any term straight off $S_n = An^2 + Bn$. **Expand it into the form $a_1 + (n - 1)d$ and hence write a_1 and d purely in terms of A and B.** You should recover exactly the coefficient rule from C7.

5. A sequence (a_n) (not assumed arithmetic) is such that its first differences $a_{n+1} - a_n$ themselves form an arithmetic sequence in n. What can be said about a_n as a function of n?

A) (a_n) must itself be arithmetic.

B) a_n is a quadratic function of n.

C) (a_n) must be geometric.

D) No general conclusion can be drawn.

Answer: B) a_n is a quadratic function of n.

Method: If the first differences $a_{n+1} - a_n$ form an AP, then the *second* differences of (a_n) are constant — and a sequence with constant second differences is exactly a quadratic function of n (the discrete analogue of a constant second derivative meaning a parabola).

> **Investigate further:** C7 in reverse: S_n was quadratic precisely because its first differences ($= a_n$) form an AP. **So push it one level further — if the *second* differences of (a_n) themselves form an AP, what kind of function of n is a_n?** State the general rule linking "kth differences constant" to the degree of a_n.

Top 5 Patterns — Worksheet #1

1. **nth-term formula: $a_n = a + (n-1)d$.** The foundation everything else builds on — always count *steps* (index differences), never raw indices. *(see A1, A2, A5, A9, A10)*
2. **Sum formula: $S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a + l)$.** Use the $(a + l)$ form whenever you're given the first and last term directly — it skips finding d entirely. *(see A3, A6, B1)*
3. **S_n is quadratic in n, with no constant term.** $S_n = \frac{d}{2}n^2 + (a - \frac{d}{2})n$ — read d and a straight off the coefficients, and extract any term via $a_n = A(2n-1) + B$. *(see C7, C8, D4, D5)*
4. **Middle-term trick for odd-length AP's.** When n is odd, $S_n = n \times$ (middle term) — no need to find a or d at all. *(see B4)*
5. **Pairing/Pigeonhole within an AP.** To force two terms to sum to a target, pair up indices by that target and count the leftover singleton — the answer is always (safe pairs) + (singletons) +1. *(see D2)*

Common Traps — Worksheet #1

1. **Forgetting the $\frac{1}{2}$ in the sum formula.** Matching $S_n = 3n^2 - n$ to $\frac{d}{2}n^2 + \cdots$ gives $d = 6$, not $d = 3$ — always solve for d from the *halved* coefficient. *(see C8)*
2. **Trusting one example for a "must be true" claim.** A single valid (a, d, n) never proves a statement true — you need a counterexample to rule one out, or a full argument to confirm one always holds. *(see C1)*
3. **Assuming every quadratic S_n-like formula is automatically a valid AP sum.** The nth-term extraction $a_n = S_n - S_{n-1}$ only makes sense when $S_0 = 0$ is consistent with the formula — check there's no stray constant term. *(see D4)*
4. **Sign errors when solving simultaneous AP equations.** A negative common difference is completely valid — don't discard a solution just because $d < 0$ looks unusual. *(see D1, B6)*
5. **Declaring a sequence arithmetic from a few terms without proving it for general n.** Checking three consecutive differences match is suggestive, not a proof — squaring an AP (C3) is exactly the case where a plausible-looking pattern breaks down algebraically. *(see C3)*

"S_n is a quadratic in n with no constant term. A stray +c means you summed a term that was never in the sequence."

Found a mistake or want to contribute a sheet?

Visit the open-source project at github.com/The-CerealDev/speed-maths