

Daily Number Theory Drill #7

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Combinatorial Number Theory · Divisor Counting · Synoptic Mastery.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition

(≤15 s each)

Primes, moduli, and basics. Pure recognition.

- Find the prime factorisation of 91.
- What is the last digit of 3^{20} ?
- Find $\gcd(144, 60)$.
- Compute $12 \times 13 \pmod{5}$.
- What is the smallest prime factor of 323?
- How many positive divisors does 28 have?
- Compute $5^3 \pmod{6}$.
- Find $x \pmod{7}$ if $4x \equiv 1 \pmod{7}$.
- What is the remainder when 10^{10} is divided by 9?
- Which is larger: 2^{30} or 3^{20} ?

Section B Manipulation Drills

(≤50 s each)

Index laws, modular equations, and divisor counting.

- Find the last two digits of 7^4 .
- How many trailing zeros are there in $20!$?
- Find the smallest positive integer n such that $120n$ is a perfect square.
- Find the largest power of 3 that divides $50!$.

5. Find the unique pair of positive integers (x, y) such that $x^2 - y^2 = 17$.
6. What is the remainder when 3^{100} is divided by 10?
7. Solve for x in the range $0 \leq x < 11$: $3x + 4 \equiv 6 \pmod{11}$.
8. Find the sum of the positive divisors of 50.
9. How many prime numbers are there strictly between 40 and 50?
10. Find the smallest three-digit positive integer that leaves a remainder of 2 when divided by 3, 4, and 5.

Section C Substitution & Structure(≤ 90 s each)*Synoptic links connecting number theory to algebra and combinatorics.*

1. Find the product of all positive integers $n < 50$ that have exactly three positive divisors. (*after SMC 2015 Q16*)
2. Find the smallest positive integer n such that n has exactly 15 positive divisors. (*after SMC 2018 Q20*)
3. How many positive divisors of $3^5 \times 5^4 \times 7^3$ are multiples of 45? (*after SMC 2021 Q19*)
4. How many positive divisors of $10!$ are perfect squares?
5. What is the greatest possible value of $\gcd(n^2 + 5, n + 2)$ for a positive integer n ?
6. Find the sum of all digits of the number $10^{15} - 2026$.
7. Find all pairs of positive integers (x, y) such that $x^3 - y^3 = 37$. (*after SMC 2023 Q1*)
8. Find the smallest positive integer x such that $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{4}$, and $x \equiv 3 \pmod{5}$.

Section D Challenge

(TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant solution — find it.

1. How many integer pairs (x, y) satisfy the equation $x^2 + xy + y^2 = 7$? (*after SMC 2023 Q10*)
2. For how many integers $n \in \{1, 2, \dots, 100\}$ is n^n a perfect square?
3. Find all pairs of positive integers (a, b) such that $a^2 - 4b^2 = 45$. (*after MAT 2021 Q1*)
4. What is the largest prime factor of $2^{16} - 1$?

5. If p and q are prime numbers such that $p^2 - 2q^2 = 1$, find the value of $p + q$. (*after SMC 2023 Q17*)

“Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised.”

Found a mistake or want to contribute a sheet?

Visit the open-source project at github.com/The-CerealDev/speed-maths