Daily Number Theory Drill #6

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: The Euclidean Step · Polynomial Division · GCD Logic.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Prime factorisation, divisibility rules, and modular arithmetic basics.

1. Find the largest prime factor of 2025.

2. What is the last digit of 3^{2026}?

3. Find the greatest common divisor of 84 and 120.

4. What is the remainder when 10^{10} is divided by 9?

5. Find the missing digit d such that the five-digit number $47d53$ is divisible by 9.

6. How many positive integer factors does 100 have?

7. Which is larger: 2^{300} or 3^{200}?

8. Evaluate $5^{2024} \pmod 4$.

9. Find the smallest positive integer n such that $12n$ is a perfect square.

10. What is the sum of the prime factors of 210?

Section B Manipulation Drills (≤50 s each)

Linear congruences, powers of primes, and prime structure.

1. Find the number of trailing zeroes of 20!.

2. Solve for x in $3x \equiv 4 \pmod 7$, with $0 < x < 7$.

3. If p and q are prime numbers such that $p^2 - q^2 = 24$, find the value of $p + q$.

4. Find the last two digits of 5^{2025}.

5. A number N leaves a remainder of 2 when divided by 3, and 3 when divided by 5. What is the smallest positive integer value of N?

6. How many perfect squares divide 3600?

7. Find the highest power of 3 dividing $27^5 \times 9^4$.

8. What is the smallest positive integer with exactly 6 positive divisors?

9. If x and y are positive integers such that $xy = 144$ and $\gcd(x, y) = 3$, find the maximum possible value of $x + y$.

10. Calculate the remainder when $1! + 2! + 3! + \cdots + 10!$ is divided by 5.

Section C Substitution & Structure (≤90 s each)

Synoptic links: use algebra and combinatorics alongside Diophantine logic.

1. Find the number of pairs of positive integers (x, y) such that $xy + x + y = 23$. *(after SMC 2018 Q16)*

2. Find the sum of all prime numbers p such that $p^2 + 2$ is also prime. *(after SMC 2016 Q20)*

3. How many 3-digit palindromes are divisible by 11? *(after SMC 2015 Q19)*

4. The product of three consecutive positive integers is equal to 336. Find the sum of these integers. *(after SMC 2012 Q14)*

5. Let N be a 4-digit number. When its digits are reversed, the new number is $N + 1089$. If the sum of the digits of N is 12, what is the maximum possible value of N? *(after SMC 2017 Q21)*

6. Find the sum of all positive integers x such that $\frac{x+11}{x-3}$ is an integer. *(after SMC 2019 Q18)*

7. Find the maximum possible value of $\gcd(7n + 15, 3n + 2)$, where n is a positive integer. *(after SMC 2013 Q21)*

8. Find the sum of all integers n such that $\frac{n^2+3n+5}{n-1}$ is an integer. *(after BMO1 2007 Q1)*

Section D Challenge (BMO1 difficulty)

Insight over computation. Uncover the structure.

1. Find all three-digit numbers abc such that $abc = a! + b! + c!$. *(after SMC 2004 Q21)*

2. Let $d_n = \gcd(n^2 + 3, n + 1)$. Find the sum of all possible values of d_n as n ranges over all positive integers. *(after SMC 2010 Q24)*

3. Find the last three digits of 2025^{2025}. *(after SMC 2008 Q24)*

4. For how many positive integers $n \leq 100$ is n^n a perfect square? *(after SMC 2001 Q22)*

5. Find the maximum value of $x + y + z$ given that x, y, z are positive integers satisfying $x \leq y \leq z$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$. *(after BMO1 2011 Q1)*

"Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths