Daily Number Theory Drill #2

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Modular Arithmetic · Remainder Equations · Parity in Modulo.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Prime factors, parity, modular remainders. Pure recognition.

1. Prime factorise 1001.

2. What is the remainder when 10^5 is divided by 11?

3. Find the last digit of 7^{100}.

4. Find the smallest positive integer k such that $12k$ is a perfect square.

5. Find the greatest common divisor of 84 and 120.

6. What is the smallest prime $p > 30$?

7. Find the remainder when 3^{10} is divided by 8.

8. Find the sum of all prime factors of 210.

9. For what single digit d is the number $53d2$ a multiple of 9?

10. Find the largest integer k such that 3^k divides 27^5.

Section B Manipulation Drills (≤50 s each)

Divisor counts, trailing zeros, modular congruence. Resist the urge to brute force.

1. Find the number of positive divisors of 144.

2. Find the smallest positive integer $x > 1$ that leaves a remainder of 1 when divided by $3, 4$, and 5.

3. Solve for the smallest positive integer x such that $5x \equiv 1 \pmod 7$.

4. Find the highest power of 5 dividing 100!.

5. Find the last two digits of 51^2.

6. Find the smallest positive integer n such that $n!$ is a multiple of 100.

7. Given that a and b are distinct positive integers with $\gcd(a, b) = 12$ and $\operatorname{lcm}(a, b) = 144$, find the minimum possible value of $a + b$.

8. What is the sum of the digits of the number $2^{10} \times 5^8$?

9. What is the remainder when $2^{20} + 3^{20}$ is divided by 5?

10. Find the number of trailing zeros in $125!$.

Section C Substitution & Structure (≤90 s each)

Look for divisibility logic, algebraic structure, and bounds before computing.

1. Find all positive integers n such that $n^2 - 4n - 5$ is a prime number. *(after SMC 2021 Q14)*

2. How many integers x are there such that $\dfrac{x^2 + 11}{x - 1}$ is an integer? *(after SMC 2019 Q20)*

3. Find the sum of all positive integers n such that $n^2 + 8n$ is a perfect square. *(after BMO1 2018 Q1)*

4. How many 3-digit numbers have the property that the product of their digits is a prime number? *(after SMC 2023 Q17)*

5. How many integers x with $1 \le x \le 100$ satisfy $x^2 \equiv 4 \pmod 5$? *(after SMC 2018 Q16)*

6. Find the number of ordered pairs of positive integers (x, y) such that $x+y = 100$ and $\gcd(x, y) = 5$. *(after SMC 2022 Q12)*

7. Find the largest integer n such that $n^3 + 100$ is divisible by $n + 10$. *(after MAT 2018 Q1)*

8. Find the sum of all positive integers $x < 20$ such that $x^2 \equiv 5 \pmod{11}$. *(after SMC 2019 Q22)*

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant solution — find it.

1. A 4-digit number A is formed by arranging the digits $1, 2, 3, 4$ in some order. What is the sum of all such possible numbers A? *(after SMC 2021 Q21)*

2. Find the number of pairs of positive integers (a, b) such that $a^2 - b^2 = 2024$. *(after BMO1 2021 Q1)*

3. Let S be the number of trailing zeros of $2025!$. What is the remainder when S is divided by 100? *(after SMC 2023 Q12)*

4. How many pairs of positive integers (x, y) with $1 \leq x, y \leq 100$ satisfy $x^2 + y^2 \equiv 0 \pmod{5}$? *(after BMO1 2017 Q2)*

5. Find the maximum value of $\gcd(n^2 + 3, n + 2)$ as n varies over all positive integers. *(after BMO1 2019 Q2)*

"Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths