Daily Number Theory Drill #1

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Core Divisibility · Parity Arguments · Alternating Sums.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Prime factors, parity, modular remainders. Pure recognition.

1. Factorise completely into primes: 2025.

2. What is the remainder when 10^6 is divided by 7?

3. Find the last digit of 3^{2024}.

4. For what single digit d is the number $73d4$ divisible by 9?

5. The positive integers a and b are distinct. Given $\gcd(a, b) = 6$ and $\text{lcm}(a, b) = 36$, find $a + b$.

6. Find the largest integer k such that 2^k divides 20!.

7. Find the remainder when 11^{10} is divided by 10.

8. Find the remainder when $2^{100} + 3^{100}$ is divided by 5.

9. What is the smallest prime factor of 1001?

10. Find the only positive integer n such that $n^2 - 1$ is a prime number.

Section B Manipulation Drills (≤50 s each)

Divisor counts, trailing zeros, modular congruence. Resist the urge to brute force.

1. Find the number of positive divisors of 360.

2. A positive integer x leaves a remainder of 3 when divided by 5, and a remainder of 5 when divided by 7. Find the smallest possible value of x.

3. What is the sum of the digits of the number $2^{2024} \times 5^{2025}$?

4. Solve for the smallest positive integer x such that $4x \equiv 5 \pmod 7$.

5. Find the last two digits of 99^2.

6. Find the greatest common divisor of $2^{12} - 1$ and $2^{18} - 1$.

7. The sum of three consecutive positive integers is a perfect cube. Find the smallest possible value for the largest of these three integers.

8. Find the smallest positive integer n such that $10n$ is a perfect square and $6n$ is a perfect cube.

9. Find the number of trailing zeros in 100!.

10. The 5-digit number $24x8y$ is divisible by 4 and by 9. Find the maximum possible value of $x + y$.

Section C Substitution & Structure

($\leq$90 s each)

Look for divisibility logic, algebraic structure, and bounds before computing.

1. Find the sum of all integers x such that $\dfrac{x^2 + 5}{x + 1}$ is an integer.

2. Find the number of ordered pairs of positive integers (a, b) such that $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{6}$.

3. For how many integers $n \in [1, 100]$ is the polynomial $x^n - 1$ divisible by $x^2 + x + 1$?

4. The 5-digit number $A34B2$, where A and B are single digits and $A \neq 0$, is a multiple of 99. Find the value of the product $A \times B$. *(after SMC 2018 Q16)*

5. The product of 24 integers is equal to 1. Their sum is 0. How many of the integers are equal to -1? *(after SMC 2015 Q11)*

6. Find the largest integer x such that 2^x divides $3^{256} - 1$.

7. Find all pairs of primes (p, q) such that $p^2 - 2q^2 = 1$.

8. A palindromic 6-digit number N is formed by placing the digits of a 3-digit number x followed by the same digits in reverse order (for example, if $x = 123$, $N = 123321$). What is the largest prime number that must be a factor of all such numbers N? *(after TMUA 2019 Paper 1 Q12)*

Section D Challenge

(TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant solution — find it.

1. Find the sum of all integers n such that $\dfrac{n^3 + 100}{n + 10}$ is an integer.

2. How many 5-digit numbers can be formed using only the digits 1, 2, and 3, such that the number is a multiple of 3?

3. Let $P(x)$ be a polynomial with integer coefficients such that $P(1) = 10$ and $P(5) = 2$. Find all possible integer roots of $P(x)$.

4. Find the number of positive divisors of the smallest positive integer n such that $\frac{n}{2}$ is a perfect square, $\frac{n}{3}$ is a perfect cube, and $\frac{n}{5}$ is a perfect fifth power.

5. Find all pairs of positive integers (x, y) such that $x^2 - y! = 2016$.

"Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths