Daily Logic Drill #5

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Spot the Flaw · Common Fallacies · Proof Critique · Valid vs. Invalid Arguments.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Fallacy literacy. Recognise the shape of an error before you have to hunt for it.

1. Name the fallacy: "If n is divisible by 6, then n is divisible by 3. $n = 15$ is divisible by 3. Therefore $n = 15$ is divisible by 6."

2. Name the fallacy: "If $x > 10$, then $x > 0$. Suppose $x \leq 10$. Therefore $x \leq 0$."

3. True or false: "If a proof of $P \implies Q$ is shown to be flawed, then $P \implies Q$ must be false."

4. A 6-line "proof" is flawed only on line 4. How many of the six lines can you actually trust?

5. True or false: "Squaring both sides of an equation is a reversible step that can never introduce a new, extraneous solution."

6. A "proof" concludes $0 = 1$. What does this tell you about the argument (assuming it is not a contradiction proof)?

7. True or false: "Verifying a universal claim $P(n)$ for $n = 1, 2, \ldots, 10$ constitutes a full proof that $P(n)$ holds for all positive integers n."

8. What is the flaw in dividing both sides of $ax = bx$ by x to conclude $a = b$?

9. True or false: "In a proof by contradiction, the final derived statement must be a strict logical contradiction (such as $0 = 1$), not merely an unusual-looking result."

10. What is wrong with concluding from $\dfrac{a}{b} = \dfrac{c}{d}$ that $a = c$ and $b = d$?

Section B Manipulation Drills (≤50 s each)

Short numbered proof attempts. Identify what, if anything, is wrong.

1. A student attempts to prove: "For all integers n, if n is odd, then n^2 is odd."
Line 1: Suppose n is odd, so $n = 2k + 1$ for some integer k.
Line 2: Then $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$.

Line 3: Since $2k^2 + 2k$ is an integer, n^2 is odd.
Is this proof valid? Justify your answer.

2. Find the flaw: "Claim: If $x^2 = 3x$, then $x = 3$." Proof: Divide both sides of $x^2 = 3x$ by x to get $x = 3$.

3. Find the flaw: "Claim: The only real solution to $\sqrt{x+7} = x - 5$ is $x = 9$." Proof: Squaring both sides gives $x + 7 = x^2 - 10x + 25$, so $x^2 - 11x + 18 = 0$, so $(x-9)(x-2) = 0$, giving $x = 9$ or $x = 2$.

4. A student argues: For every integer n, $6\left(\frac{4n+1}{2} - \frac{n-3}{2}\right)$ is a multiple of 3. *(after TMUA 2020 Paper 2 Q3)*
Line 1: $6\left(\frac{4n+1}{2} - \frac{n-3}{2}\right) = 3\left(2\left(\frac{4n+1}{2} - \frac{n-3}{2}\right)\right)$.
Line 2: $= 3(4n + 1 - n - 3)$.
Line 3: $= 3(3n - 2)$.
Line 4: which is always a multiple of 3.
On which line does the argument first contain an error, and what exactly is that error?

5. Find the flaw: "Claim: For every integer n, $n^2 + n + 2$ is even." Proof: If n is even, $n = 2k$, so $n^2 + n + 2 = 4k^2 + 2k + 2 = 2(2k^2 + k + 1)$, which is even. Hence $n^2 + n + 2$ is even for every integer n.

6. Find the flaw: "Claim: For real x, $x^2 = 9 \iff x = 3$." Proof: If $x = 3$, then $x^2 = 9$. This proves the biconditional.

7. Find the flaw: "Claim: For every positive integer n, the n-th Fibonacci number F_n (with $F_1 = F_2 = 1$) is either equal to 1 or prime." Proof: $F_3 = 2$, $F_4 = 3$, $F_5 = 5$ — all prime. Since the pattern holds for $n = 3, 4, 5$, the claim is proved. What is the smallest counterexample?

8. A student wants to prove: "If n is not divisible by 3, then n^2 is not divisible by 3." Instead, they show: "If n is divisible by 3, then n^2 is divisible by 3." Explain why this does not prove the original claim.

9. Find the flaw: "Claim: For all integers a, b, if a and b are both even, then $a + b$ is even." Proof: Since $a + b$ is even, $a + b = 2m$ for some integer m. Since a, b are even, $a = 2j, b = 2k$, so $2j + 2k = 2m$, i.e. $j + k = m$, which is true since m is an integer. Hence $a + b$ is even.

10. A student is told: "It is not the case that every student in the class passed the exam." They conclude: "Therefore, every student in the class failed the exam." What is wrong with this conclusion?

Section C Substitution & Structure ($\leq$90 s each)

Tricky MCQs. Some of these proofs are correct — don't assume a flaw exists.

1. Consider the claim: "For every positive integer n, $2n^2 + 2n + 1$ is prime." *(after TMUA 2022 Paper 2 Q7)*
Attempted proof:

I. Consider the quadratic $f(x) = 2x^2 + 2x + 1$.
II. The discriminant of $2x^2 + 2x + 1$ is $2^2 - 4(2)(1) = -4 < 0$.
III. Since the discriminant is negative, $2x^2 + 2x + 1$ cannot be factorised into two linear factors with integer coefficients.
IV. Therefore, for every positive integer n, the integer $2n^2 + 2n + 1$ cannot be factorised, so it is prime.
Which of the following best evaluates this argument?

A) The proof is completely valid, and the claim is true.

B) The proof is invalid; the error occurs on line II.

C) The proof is invalid; the error occurs on line III.

D) The proof is invalid; the error occurs on line IV — irreducibility of the polynomial does not imply primality of its integer values ($n = 3$ gives $2(9) + 6 + 1 = 25 = 5^2$).

E) The proof is invalid; the error occurs on line I.

2. Let P: "$x^2 - 5x + 6 = 0$" and Q: "$x = 2$". A student argues: "Since $Q \implies P$ is true (substituting $x = 2$ gives $4 - 10 + 6 = 0$), and P and Q are 'clearly related', we conclude $P \iff Q$." Which of the following identifies the flaw?

A) $Q \implies P$ is actually false.

B) $P \implies Q$ is false, since $x = 3$ also satisfies P but not Q; asserting $P \implies Q$ without proof is the error.

C) There is no flaw; the conclusion is correct.

D) P and Q are not comparable statements.

E) The flaw is that Q should be shown to imply $\neg P$.

3. A student attempts to prove: "For every integer n, $n^2 + n$ is divisible by 2." Proof: Suppose n is even, so $n = 2k$. Then $n^2 + n = 4k^2 + 2k = 2(2k^2 + k)$, which is divisible by 2. Hence the claim is proved. Which of the following is the most accurate evaluation?

A) The proof is complete and correct.

B) The proof is incomplete: only the even case was considered; the odd case must also be checked (though the claim does turn out to be true for odd n too).

C) The proof is invalid because $n = 2k$ is an unjustified assumption.

D) The claim itself is false; $n = 3$ is a counterexample.

E) The proof incorrectly divides by 2.

4. A student attempts to prove: "No three consecutive odd positive integers can all be prime." *(after TMUA 2023 Paper 2 Q4)*
Line I: Let the three consecutive odd integers be $n - 2, n, n + 2$.
Line II: Exactly one of $n - 2, n, n + 2$ is divisible by 3, for any integer n.
Line III: A positive integer divisible by 3 is composite.
Line IV: So one of the three integers is composite, and therefore no three consecutive odd positive integers can all be prime.
On which line does the argument first break down?

A) Line I

B) Line II

C) Line III — a positive multiple of 3 need not be composite, since 3 itself is prime; e.g. $n-2=3, n=5, n+2=7$ are three consecutive odd primes.

D) Line IV

E) The argument is fully correct.

5. A student "proves": "For every positive integer n, there exists an integer m with $m > n$." They conclude: "Therefore, there exists an integer M that is greater than every positive integer n." Which of the following identifies the flaw?

A) No flaw; both statements are equivalent.

B) The student has swapped the order of quantifiers: $\forall n\, \exists m\, (m > n)$ does not imply $\exists M\, \forall n\, (M > n)$.

C) The flaw is that m must be positive.

D) The flaw is that no such m exists at all.

E) The two statements are both false.

6. Claim: "If a quadrilateral is a rhombus, then its diagonals are perpendicular." A student, wanting to test the converse, argues: "The diagonals of a square are perpendicular, and a square is a rhombus, so this confirms the converse: if a quadrilateral's diagonals are perpendicular, it is a rhombus." What is the flaw in this reasoning?

A) A square is not a rhombus.

B) One supporting example does not prove a universal converse statement — a kite also has perpendicular diagonals but need not be a rhombus.

C) The diagonals of a square are not actually perpendicular.

D) The original claim about rhombi is false.

E) There is no flaw.

7. A student is asked whether the following claim $(*)$ is true: "For every pair of real numbers x, y, $|x+y| < |x| + |y|$." *(after TMUA 2020 Paper 2 Q8)*
The student answers: "$(*)$ is false — take $x=-2, y=-5$. Then $|x+y| = |-7| = 7$ and $|x|+|y| = 2+5 = 7$, so the required strict inequality $7 < 7$ fails, which disproves $(*)$."
Assess this answer.

A) The claim $(*)$ is actually true, so the student's disproof must contain a mistake somewhere.

B) The chosen values $x=-2, y=-5$ do not disprove $(*)$, because a valid counterexample needs x and y to have opposite signs.

C) A single pair of values is never enough evidence against a "for every" claim; more pairs would need to be checked before $(*)$ can be rejected.

D) The student's reasoning is sound as it stands: $(*)$ is genuinely false, and one correctly-checked pair is sufficient to establish that.

8. Claim: "Having four equal sides is a necessary and sufficient condition for a quadrilateral to be a square." A student "verifies" this by checking that every square has four equal sides. What is the flaw in this verification?

A) Squares do not have four equal sides.

B) The student has only checked necessity (square $\implies$ four equal sides), not sufficiency (four equal sides $\implies$ square) — a rhombus that is not a square is a counterexample to sufficiency.

C) The student has only checked sufficiency, not necessity.

D) There is no flaw.

E) Both directions were checked correctly.

Section D Challenge (TMUA / SMC difficulty)

Longer critiques. Some proofs here are entirely valid — read every line before you judge.

1. A theorem and a flawed attempt to prove it: *(after TMUA 2021 Paper 2 Q11)*
Theorem: If s, p are real numbers with $s^2 \geq 4p$, then there exist real numbers u, v such that $u + v = s$ and $uv = p$.
Attempt:
I. Suppose u, v are real numbers with $u + v = s$ and $uv = p$.
II. Then $(u - v)^2 \geq 0$, so $u^2 - 2uv + v^2 \geq 0$, i.e. $(u + v)^2 - 4uv \geq 0$.
III. Substituting, $s^2 - 4p \geq 0$.
IV. This proves $s^2 \geq 4p$, so the theorem is shown.
Which of the following best describes this argument?

A) It is completely correct.

B) It is incorrect, but its logic runs backward: the student has shown that $s^2 \geq 4p$ *follows from* the existence of u, v, which is the converse of what is required. The theorem needs u, v to be *constructed from* $s^2 \geq 4p$ (e.g. as the roots of $t^2 - st + p = 0$).

C) It is incorrect because line II is false.

D) It is incorrect because line III is false.

E) There is no flaw; assuming u, v exist first is a standard, valid proof technique here.

2. A student claims: "For every positive integer n, if $n^2 - n + 11$ is prime, then $n^2 + n + 11$ is also prime." The student checks $n = 1, \ldots, 9$, finds the claim holds in every case, and concludes the claim is proved.
(a) Explain why this does not constitute a proof.
(b) Find the smallest positive integer n for which the claim fails.

3. A proof of the following claim is given: "For all integers a, b: if ab is even, then a is even or b is even."
Proof attempt: Suppose, for contradiction, that ab is even but a and b are both odd. Write $a = 2j + 1, b = 2k + 1$. Then $ab = 4jk + 2j + 2k + 1 = 2(2jk + j + k) + 1$, which is odd. This contradicts ab being even. Hence a is even or b is even.
Which of the following statements about this proof is/are true?
I. The proof is a valid proof by contradiction.
II. The proof could equally well be presented as a direct proof by contrapositive.
III. The proof contains an algebraic error in expanding $(2j + 1)(2k + 1)$.

A) I only

B) II only

C) III only

D) I and II only

E) I, II and III

F) none

4. The following argument is given for the claim "every odd prime greater than 3 can be written as $6k \pm 1$ for some positive integer k":
Line 1: Every integer can be written as $6k, 6k+1, 6k+2, 6k+3, 6k+4$, or $6k+5$ for some integer $k \geq 0$.
Line 2: $6k, 6k+2, 6k+4$ are all even, so none can be an odd prime greater than 3.
Line 3: $6k+3 = 3(2k+1)$ is divisible by 3, so cannot be a prime greater than 3.
Line 4: This leaves only $6k+1$ and $6k+5 = 6(k+1)-1$, i.e. of the form $6k'-1$. So every odd prime greater than 3 has the form $6k \pm 1$.
Which of the following is true?

A) The argument is fully valid, and correctly establishes the claim exactly as stated.

B) The argument shows only that every prime > 3 has the form $6k \pm 1$ (necessity), not that every number of that form is prime (sufficiency) — but the claim as stated only asserts the former, so the proof is complete.

C) There is a genuine gap: line 2 incorrectly dismisses $6k+4$.

D) There is a genuine gap: line 3 is false.

E) The proof is invalid because it never explicitly rules out $6k$ itself.

5. A student attempts to prove: "For all positive integers $n \geq 2$, $n^2 - 1$ is composite."
Line 1: $n^2 - 1 = (n-1)(n+1)$.
Line 2: For $n \geq 2$, $n - 1 \geq 1$ and $n + 1 \geq 3$.
Line 3: Since $n^2 - 1$ is written as a product of two integers each at least 1, it is composite.
Line 4: Hence $n^2 - 1$ is composite for all $n \geq 2$.
(a) Identify the line containing the first logical gap.
(b) Give the smallest $n \geq 2$ for which the conclusion in Line 4 is actually false, and prove that this is the *only* value of $n \geq 2$ for which it fails — i.e. that the conclusion is genuinely correct for every $n \geq 3$.

"Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised."

Found a mistake or want to contribute a sheet?
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