Daily Logic Drill #3

Sheet by David Akinyele-Aje

Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Direct Proof · Proof by Contrapositive.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Short, immediate proofs — direct and by contrapositive, paired so you see both routes to the same fact.

1. Prove directly: "If n is even, then n^2 is even."

2. Prove directly: "If n is odd, then n^2 is odd."

3. Prove directly: "If a and b are both even, then $a+b$ is even."

4. State the contrapositive of "If n is a multiple of 6, then n is even." Prove the contrapositive directly.

5. Prove directly: "If a and b are both odd, then ab is odd."

6. True or false: "To prove 'if P then Q' by contrapositive, you should assume Q is false and show P is false."

7. Prove by contrapositive: "If n^2 is odd, then n is odd."

8. True or false: "A proof by contrapositive is a different technique from proving the converse."

9. Prove directly: "If $x > 1$, then $x^2 > 1$."

10. Prove by contrapositive: "If $x^2 \leq 1$, then $x \leq 1$."

Section B Manipulation Drills (≤50 s each)

Slightly longer proofs, and judgement calls about which route is more natural.

1. Prove directly: "If n is a multiple of 4, then n^2 is a multiple of 16."

2. Prove by contrapositive: "If n^2 is not a multiple of 3, then n is not a multiple of 3."

3. Which proof route is more natural for "If n^2 is even, then n is even" — direct or contrapositive? Prove it using that route.

4. Prove directly: "If $a > b > 0$, then $a^2 > b^2$."

5. Prove by contrapositive: "If $x^2 - 4x + 3 \neq 0$, then $x \neq 1$ and $x \neq 3$."

6. Prove directly: "If n is a multiple of 3, then n^3 is a multiple of 27."

7. State and prove the contrapositive of "If xy is irrational, then x is irrational or y is irrational."

8. Prove directly: "If n ends in the digit 5, then n^2 ends in the digit 5."

9. Prove by contrapositive: "If n is not a multiple of 4, then n^2 is not a multiple of 16."

10. True or false: "Every proof by contrapositive can be rewritten as a direct proof of the original statement."

Section C Substitution & Structure ($\leq$90 s each)

Multi-step proofs, and your first "which condition is sufficient" MCQ.

1. For non-zero real numbers x, y, which one of the following is sufficient to conclude that $x < y$?

A) $x^2 < y^2$

B) $y^2 < x^2$

C) $|x| < |y|$

D) $|y| < |x|$

E) $x^5 < y^5$

F) $y^5 < x^5$

2. Prove by contrapositive: "If n^3 is even, then n is even."

3. Prove directly: "If $x, y > 0$ and $x^2 > y^2$, then $x > y$."

4. Prove directly: "If n is a multiple of 3 but not a multiple of 9, then n^2 is a multiple of 9 but not a multiple of 81."

5. Prove by contrapositive: "If x and y are real numbers with xy irrational, then $x \neq 0$ and $y \neq 0$."

6. Prove by contrapositive: "If n is an integer such that $2n^2 + 1$ is a multiple of 3, then n is not a multiple of 3."

7. Prove directly: "If n is a positive integer that is not a multiple of 3, then $n^2 \equiv 1 \pmod 3$."

8. True or false, with justification: "A statement with no counterexamples found after checking 100 cases has therefore been proven true by direct proof."

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant solution — find it.

1. A region is defined by the inequalities $x + y > 10$ and $x - y > 2$. Consider the three statements: I. $x > 6$ II. $y > 4$ III. $(x + y)(x - y) > 20$. Which of the above statements is/are true for every point in the region?

A) none

B) I only

C) II only

D) III only

E) I and II only

F) I and III only

G) II and III only

H) I, II and III

2. Prove by contradiction: "There is no pair of positive integers (m, n) with $m^2 - n^2 = 2018$."

3. Prove directly: "If $n > 4$ is such that both $n - 1$ and $n + 1$ are prime, then n is a multiple of 6."

4. Prove directly: "If x and y are positive reals with $x \neq y$, then $\dfrac{x + y}{2} > \sqrt{xy}$."

5. Statement T: "If n is a positive integer such that n^2 is a multiple of 6, then n is a multiple of 6." (a) Is T true? Prove or disprove. (b) Contrast with the fact that "n^2 is a multiple of 4" does *not* force "n is a multiple of 4" (witnessed by $n = 6$). What property of 6, that 4 lacks, makes T true?

"Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths