

Daily Logic Drill #2

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Implication, Converse, Inverse & Contrapositive · Necessary vs Sufficient.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition

(≤15 s each)

Construct a converse/contrapositive/inverse, or classify a condition. Fluency, not first contact.

1. State the converse of: “If n is a multiple of 6, then n is a multiple of 3.”
2. State the contrapositive of: “If n is a multiple of 6, then n is a multiple of 3.”
3. True or false: “ $x = 3$ ” is sufficient for “ $x^2 = 9$ ”.
4. True or false: “ $x = 3$ ” is necessary for “ $x^2 = 9$ ”.
5. Is “ n is a multiple of 4” necessary, sufficient, or neither, for “ n is a multiple of 2”?
6. State the inverse of: “If n is prime, then n is odd or $n = 2$.”
7. True or false: “The contrapositive of a true statement is always true.”
8. True or false: “The converse of a true statement is always true.”
9. Negate: “If n is even, then n^2 is even.”
10. True or false: “ n is a multiple of 12” is sufficient for “ n is a multiple of 4”.

Section B Manipulation Drills

(≤50 s each)

Build converse/contrapositive/inverse together, and classify with justification.

1. State both the converse and contrapositive of “If $x > 2$, then $x^2 > 4$.” Determine the truth of each.
2. Classify “ $n = 6$ ” as necessary, sufficient, both, or neither, for “ n is a multiple of 3”.
3. Classify “ n is divisible by 6” as necessary, sufficient, both, or neither, for “ n is divisible by 2 and by 3”.
4. State the contrapositive of “If a and b are both even, then $a + b$ is even.” Is it true?

5. True or false: “If $P \Rightarrow Q$ and $Q \Rightarrow P$ are both true, then P and Q are necessary and sufficient for each other.”
6. Given that “ x is rational” is sufficient for “ x^2 is rational”, state the converse. Is the converse true?
7. State the inverse and the converse of “If n is prime, then n is odd or $n = 2$.” Are they logically equivalent to each other?
8. True or false: “ $a = b$ ” is necessary and sufficient for “ $a^2 = b^2$ ”.
9. Is “ $x^2 > 4$ ” necessary, sufficient, both, or neither, for “ $x > 2$ ”?
10. State the converse of “If n is a multiple of 9, then n is a multiple of 3.” Is the converse true?

Section C Substitution & Structure(≤ 90 s each)*Chain necessary/sufficient claims, and use the inverse-converse link.*

1. For real numbers x, y , the condition “ $x + y$ is an integer and $x - y$ is an integer” is:
 - A) necessary but not sufficient for “ x and y are both integers”
 - B) sufficient but not necessary for “ x and y are both integers”
 - C) necessary and sufficient for “ x and y are both integers”
 - D) neither necessary nor sufficient for “ x and y are both integers”
2. Given that P is necessary and sufficient for Q , and Q is necessary and sufficient for R , prove that P is necessary and sufficient for R .
3. Classify “a triangle has all three altitudes of equal length” as a condition for “the triangle is equilateral”. Justify using the fact that an altitude to side a has length $\frac{2 \times \text{Area}}{a}$.
4. For positive integers a, b, n , suppose you’re told only that n divides a , or n divides b (possibly both). Does this pin down whether n divides the product ab ? And working the other way: does knowing n divides ab pin down whether n divides a or n divides b ? (after TMUA 2019 Paper 2 Q5)
 - A) “ n divides a or n divides b ” is necessary but not sufficient for “ n divides ab ”
 - B) “ n divides a or n divides b ” is sufficient but not necessary for “ n divides ab ”
 - C) “ n divides a or n divides b ” is necessary and sufficient for “ n divides ab ”
 - D) “ n divides a or n divides b ” is neither necessary nor sufficient for “ n divides ab ”
5. Statement S : “If n is a multiple of 8, then n is a multiple of 2.” Determine whether S ’s converse is true by testing S ’s inverse instead. State the inverse, and your conclusion about the converse.
6. Classify “ n^2 is even” as a condition for “ n is even”.
 - A) necessary but not sufficient

- B) sufficient but not necessary
 - C) necessary and sufficient
 - D) neither necessary nor sufficient
7. Statement S is true, and S 's inverse is also true. What can you conclude about S 's converse? Justify.
8. State the contrapositive of "If a and b are both irrational, then ab is irrational." Is the original statement true? Give a counterexample if false.

Section D Challenge

(TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant solution — find it.

1. Let $f(x) = x^2 + bx + c$ with $f(0) = c$. Let P be the statement: "if $c < 0$, then f has two distinct real roots." Which of the following must be true? (after TMUA 2022 Paper 2 Q5)
- I. P
 - II. The converse of P
 - III. The contrapositive of P
- A) none of them
 - B) I only
 - C) II only
 - D) III only
 - E) I and II only
 - F) I and III only
 - G) II and III only
 - H) I, II and III
2. Consider a finite list of n real numbers (repeats allowed), sorted into non-decreasing order. Let P be the statement " n is odd", and let Q be the statement "the middle value of the sorted list equals one of the list's own entries". P is: (after TMUA 2022 Paper 2 Q6)
- A) necessary but not sufficient for Q
 - B) sufficient but not necessary for Q
 - C) necessary and sufficient for Q
 - D) neither necessary nor sufficient for Q
3. For a positive integer n , let P be " n leaves remainder 1 when divided by 4" and Q be " n^2 leaves remainder 1 when divided by 8." P is:
- A) necessary but not sufficient for Q
 - B) sufficient but not necessary for Q
 - C) necessary and sufficient for Q

D) neither necessary nor sufficient for Q

4. Statement S : “If a quadrilateral has four equal sides, then its diagonals are perpendicular.” (a) Is S true? (b) State S ’s converse. Is the converse true?
5. Statement S : “If n is a positive integer such that $n^2 - 1$ is prime, then $n = 2$.” (a) Is S true? Prove or disprove. (b) State S ’s contrapositive. (c) Is “ $n^2 - 1$ is prime” necessary and sufficient for “ $n = 2$ ”?

“Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised.”

Found a mistake or want to contribute a sheet?

Visit the open-source project at github.com/The-CerealDev/speed-maths