Daily Logic Drill #1

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Statements & Quantifiers (∀/∃) · Negating Compound Statements · Negating Quantified Statements · Order of Quantifiers.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Apply a negation rule immediately — fluency, not first contact.

1. Negate: "For all integers n, $n^2 \geq 0$."

2. Negate: "There exists a real number x such that $x^2 = -1$."

3. Negate: "n is even and n is prime."

4. Negate: "n is a multiple of 4 or n is a multiple of 6."

5. True or false: "The negation of 'for all x, $P(x)$' is 'for all x, not $P(x)$'."

6. True or false: "not(P and Q) is the same as (not P) or (not Q)."

7. Negate: "$x > 3$ and $x < 10$."

8. Is the statement "some prime numbers are even" a ∀-statement or a ∃-statement in form?

9. True or false: "$\exists x\, \forall y : x + y = 0$" and "$\forall y\, \exists x : x + y = 0$" (both over the reals) mean the same thing.

10. Negate: "n is prime or $n = 1$."

Section B Manipulation Drills (≤50 s each)

Negate nested and multi-part statements; start noticing where quantifier order changes meaning.

1. Consider the statement about Fred: "Every day next week, Fred will do at least one maths problem." State the negation. *(after TMUA Specimen Paper 2 Q9)*

2. True or false: "not(P or Q or R) is the same as (not P) and (not Q) and (not R)."

3. Negate: "n is a multiple of 2 and n is a multiple of 3 and n is a multiple of 5."

4. Negate: "For every real $x > 0$, there exists a real $y > 0$ with $y < x$."

5. True or false: "'$\forall n\, \exists m : m > n$' and '$\exists m\, \forall n : m > n$' (both over the integers) mean the same thing."

6. Negate the nested statement: "There exists a prime p such that for every prime $q > p$, q is odd." Is the original statement true or false?

7. Negate: "n is a multiple of 3, or (n is a multiple of 2 and n is a multiple of 5)."

8. True or false: "not(A and (B or C)) is the same as (not A) or ((not B) and (not C))."

9. Negate: "x is rational, or (x is irrational and $x > 0$)."

10. Negate: "For every prime $p > 2$, p is odd." Is the negation true or false?

Section C Substitution & Structure (≤90 s each)

Multi-step negations, nested quantifiers, and your first ordering-of-quantifiers MCQ.

1. For real x and positive integer n, consider: I. For all x and for all n, $x^2 < n$. II. For all x, there exists n such that $x^2 < n$. III. There exists x such that for all n, $x^2 < n$. Which of I, II, III are true? *(after TMUA 2022 Paper 2 Q10)*

2. Determine the complete set of real values of k for which the following is true: "For all real x, if $x < k$, then $x^2 > k$."

A) no real numbers

B) $k < 0$

C) $k \leq 0$

D) $k \leq 1$

E) all real numbers

F) $k > 0$

3. Which is the correct negation of "For all primes p, p is odd or $p = 2$"?

A) There exists a prime p such that p is odd or $p = 2$.

B) There exists a prime p such that p is not odd and $p \neq 2$.

C) For all primes p, p is not odd and $p \neq 2$.

D) There exists a prime p such that p is not odd or $p \neq 2$.

4. Negate: "There exists a positive integer n such that $n^2 + n + 1$ is even." Is the negation true or false?

5. Negate: "There exists a positive integer n such that for every positive integer $k \leq 10$, k divides n." Is the original statement true or false?

6. Consider the statement about a real number x: "There exists a real number y such that $xy-x+y$ is positive." For how many real values of x is this true? *(after TMUA 2022 Paper 2 Q13)*

A) no values of x

B) exactly one value of x

C) all except exactly one value of x

D) all except exactly two values of x

E) all values of x

7. Negate: "There exists a real number x such that for every real number y, $xy = y$." Is the original statement true or false?

8. Negate: "For every positive integer n, there exists a positive integer $m < n$ that is prime." Consider the cases $n = 1$ and $n = 2$ carefully. Is the original statement true or false?

Section D Challenge (TMUA / SMC difficulty)

Insight over computation. Each question has a short, elegant solution — find it.

1. Consider the statement: "For every finite sequence of positive integers, there exists a positive integer m such that every term of the sequence is at most m." Which of the following is the correct negation?

A) There exists a finite sequence of positive integers such that, for every positive integer m, some term exceeds m.

B) There exists a finite sequence such that, for every positive integer m, every term is at most m.

C) For every finite sequence, there exists a positive integer m such that some term exceeds m.

D) There exists an infinite sequence such that no positive integer bounds all terms.

2. Determine, with brief justification, whether the following is true or false: "For every positive real number ε, there exists a positive integer N such that for every integer $n \geq N$, $\dfrac{1}{n} < \varepsilon$."

3. A selection S of n terms is taken from the arithmetic sequence $2, 5, 8, 11, \ldots, 92$. What is the smallest value of n for which S must contain two distinct terms summing to 94? *(after TMUA 2022 Paper 2 Q8)*

A) 15

B) 16

C) 17

D) 18

E) 24

F) 31

4. Statement $(*)$: "For every finite set of primes $\{p_1, \ldots, p_k\}$, there exists a prime not in the set." (a) Negate $(*)$. (b) Is $(*)$ true or false? Justify briefly.

5. Let $f : \mathbb{R} \to \mathbb{R}$. Consider: I. "For every x, there exists y such that $f(y) = x$." II. "There exists y such that for every x, $f(y) = x$." (a) Are I and II logically equivalent? (b) Give a concrete function f for which I is true but II is false.

"Speed comes from recognition, not from rushing. Every shortcut is a pattern you have internalised."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths