Daily Combinatorics Drill #4

Sheet by David Akinyele-Aje

Section	**Topic**	**Questions**	**Target time**
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Binomial Theorem · Pascal's Rule · Coefficient Extraction · Substituting Values into Expansions.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Coefficients on sight. One binomial fact per question.

1. Find the coefficient of x^2 in $(1+x)^5$.

2. Find the coefficient of x in $(x+2)^3$.

3. Write down row 5 of Pascal's triangle.

4. Find the coefficient of x^3y^2 in $(x+y)^5$.

5. What is the sum of all the coefficients in the expansion of $(1+x)^7$?

6. Find the constant term of $\left(x+\dfrac{1}{x}\right)^4$.

7. Express $\binom{9}{4}+\binom{9}{5}$ as a single binomial coefficient.

8. Evaluate: $\binom{6}{0}-\binom{6}{1}+\binom{6}{2}-\cdots+\binom{6}{6}$.

9. True or false: the coefficient of x^2 in $(1-x)^{10}$ is negative.

10. Write down the largest coefficient in the expansion of $(1+x)^4$.

Section B Manipulation Drills (≤50 s each)

General terms and clean extractions. Write $\binom{n}{k}a^{n-k}b^k$ and read off k.

1. Find the coefficient of x^3 in $(2+3x)^5$.

2. Find the coefficient of x^5 in $(1+2x)^4(1+x)^3$.

3. Find the term independent of x in $\left(2x + \frac{1}{x^2}\right)^6$.

4. The coefficient of x^2 in $(1+x)^n$ is 66. Find n.

5. Find the coefficient of x^6 in $\left(x^2 + \frac{1}{x}\right)^6$.

6. Find the coefficient of x^2 in $(1+2x)^4(1-x)^2$.

7. Use the binomial theorem to evaluate 11^3 and 9^3 without long multiplication.

8. The coefficient of x^3 in $(1+kx)^6$ is 160. Find k.

9. Find the value of n for which $\binom{n}{2} = \binom{n}{3}$.

10. Find the coefficient of a^2b^2c in $(a+b+c)^5$. *(cf. TMUA 2020 P1 Q13)*

Section C Substitution & Structure

($\leq$90 s each)

Choose what to substitute — $x = 1$, $x = -1$, or a cunning value — before expanding anything.

1. Evaluate: $\binom{10}{0} + \binom{10}{2} + \binom{10}{4} + \cdots + \binom{10}{10}$.

2. Which coefficient in the expansion of $(1+x)^8$ is the greatest, and what is its value?

3. Use the first three terms of a binomial expansion to estimate 1.01^{10} to 4 decimal places.

4. In the expansion of $(1+3x)^n$, the coefficient of x^5 is exactly three times the coefficient of x^4. Find n.

5. Find the coefficient of x^4 in $(1+x)^5(1-2x)^3$. *(cf. MAT 2023 Q1F)*

6. Evaluate $\sum_{k=0}^{5} k\binom{5}{k}$.

7. By substituting a suitable value into an expansion, evaluate $\sum_{k=0}^{6} \binom{6}{k} 2^k$.

8. Prove Pascal's rule $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ by a committee argument, and use it to generate row 7 of Pascal's triangle from row 6.

Section D Challenge

(SMC / BMO1 difficulty)

Proof enters the drill. Write your arguments in full sentences — BMO1 marks the writing, not the answer.

1. Prove that $\binom{2n}{n}$ is even for every integer $n \geq 1$.

2. Show that $\sum_{k=0}^{10} \binom{10}{k}^2 = \binom{20}{10}$, by counting one scenario in two ways.

3. Prove that exactly four of the coefficients $\binom{12}{k}$, $0 \leq k \leq 12$, are odd. *(cf. TMUA 2023 P1 Q6)*

4. Show that $\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \cdots + \binom{9}{2} = \binom{10}{3}$, and state its value.

5. Let p be a prime and $0 < k < p$. Prove that $\binom{p}{k}$ is divisible by p.

"An identity is a story told twice — once for each side of the equals sign."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths