Daily Algebra Drill #4

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Geometric series identity · Polynomial remainder shortcut · Parity and mod arguments · AM–GM under substitution.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

No expansion. Recognise the pattern, write the answer.

1. Evaluate without expanding: 201×199.

2. Factorise: $x^2 - x - 42$.

3. Simplify: $1 + x + x^2 + x^3 + x^4$ when $x = -1$.

4. Given $p + q = 6$ and $pq = 7$, find $p^3 + q^3$.

5. Without long division, find the remainder when $f(x) = x^3 + 2x - 3$ is divided by $(x - 1)$.

6. Factorise completely: $2x^3 - 18x$.

7. Simplify: $\dfrac{3^{n+1} - 3^{n-1}}{3^n}$.

8. Given $a - b = 3$ and $a^2 + b^2 = 29$, find ab.

9. Factorise: $x^4 - 1$ completely over $\mathbb{R}$.

10. Simplify: $\dfrac{n!}{(n-2)!}$.

Section B Manipulation Drills (≤50 s each)

Fractions, structure, identities. Minimum working.

1. Using the geometric series formula, simplify: $\dfrac{x^5 - 1}{x - 1}$.

2. Find the remainder when $g(x) = x^{10} - 3x^5 + 2$ is divided by $(x - 1)$ and by $(x + 1)$.

3. Simplify: $\dfrac{ab}{a-b} - \dfrac{a^2 b}{a^2 - b^2}$.

4. Factorise: $x^6 - y^6$ completely over $\mathbb{Z}$.

5. The roots of $3x^2 + 5x - 2 = 0$ are α and β. Find $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

6. Simplify: $\left(1 - \dfrac{1}{x^2}\right)(x^2 + x)$.

7. Given $a + b + c = 5$, $ab + bc + ca = 6$, find $a^2 + b^2 + c^2$.

8. Rearrange for y: $\dfrac{2y+1}{y-3} = x$.

9. Show that $4n^2 - 1 = (2n-1)(2n+1)$, hence prove that $4n^2 - 1$ is never prime for integers $n \geq 2$.

10. Given α, β satisfy $\alpha + \beta = 4$ and $\alpha\beta = 1$, find a quadratic with integer coefficients whose roots are $\alpha - \frac{1}{\alpha}$ and $\beta - \frac{1}{\beta}$.

Section C Substitution & Structure (≤90 s each)

One clean substitution is worth ten lines of algebra.

1. Solve: $4^x - 5 \cdot 2^x + 4 = 0$.

2. Given $t = x + \dfrac{1}{x}$, express $x^3 + \dfrac{1}{x^3}$ in terms of t.

3. Find the minimum value of $3x + \dfrac{27}{x}$ for $x > 0$, and the value of x at which it is achieved.

4. Solve for real x: $\sqrt{2x-1} + \sqrt{x-1} = 2$.

5. The expression $(x+1)(x+3)(x+5)(x+7) + k$ is a perfect square for all x when k takes a specific value. Find k.

6. Prove that $(a^2 + b^2)(c^2 + d^2) \geq (ac + bd)^2$ for all reals a, b, c, d. What is the condition for equality?

7. If $u = x + y$ and $v = x - y$, express $x^4 + y^4$ purely in terms of u and v.

8. Solve: $x^4 - 7x^2 + 12 = 0$.

Section D Challenge (TMUA / SMC difficulty)

No brute force. Each question rewards one key observation.

1. For all integers n, prove that n^2 is either divisible by 4 or leaves remainder 1 when divided by 4. Hence prove that $n^2 + 2$ is never divisible by 4.

2. Find all real solutions to $x^4 - 4x^3 + 4x^2 - 1 = 0$. (*Look for a way to write the left-hand side as a product of two quadratics.*)

3. A sequence is defined by $a_1 = 1$ and $a_{n+1} = \dfrac{a_n}{a_n + 2}$. Find a closed form for a_n and hence compute a_{100}.

4. Find the value of

$$\left(1 - \tfrac{1}{4}\right)\left(1 - \tfrac{1}{9}\right)\left(1 - \tfrac{1}{16}\right)\cdots\left(1 - \tfrac{1}{n^2}\right).$$

Express your answer in closed form and state its limit as $n \to \infty$.

5. The polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$ satisfies $p(1) = 10$, $p(2) = 20$, $p(3) = 30$, $p(4) = 40$. Find $p(12) - 12p(0)$ without finding a, b, c, d.

"The person who can solve in 5 minutes what others solve in 50 has not worked 10 times harder — they have seen something different."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths