Daily Algebra Drill #2

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Section	Topic	Questions	Target time
A	Rapid Recognition	10	2 min 30 sec
B	Manipulation Drills	10	8 min
C	Substitution & Structure	8	10 min
D	Challenge	5	15 min

New toolkit today: Vieta's formulas for cubics · Nested surds · Telescoping products · Lagrange/Brahmagupta identity · Factor theorem · Cyclic sums under constraints.

Attempt each question before checking answers. Time yourself. No calculators.

Section A Rapid Recognition (≤15 s each)

Factorise, simplify, or evaluate instantly. No expansion.

1. Factorise completely: $x^3 - 27$.

2. Without expanding, evaluate $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$.

3. Without a calculator, evaluate $999^2 - 1$.

4. Given $a - b = 4$ and $ab = 5$, find $a^2 + b^2$.

5. Expand and simplify: $(x + y)^3 - x^3 - y^3$.

6. Factorise completely: $3x^2 - 75$.

7. Rationalise the denominator: $\dfrac{1}{\sqrt{3} + \sqrt{2}}$.

8. Factorise: $x^2(x - 1) - (x - 1)$.

9. The roots of $x^2 - 5x + 3 = 0$ are p and q. Find $p^2 + q^2$.

10. Simplify: $\dfrac{2^{n+3} - 2^n}{2^n}$.

Section B Manipulation Drills (≤50 s each)

Fractions, rearranging, hidden factorisations. Elegance over brute force.

1. Simplify: $\dfrac{x}{x^2 - 1} + \dfrac{1}{x - 1}$.

2. Factorise: $x^4 - x^2 - 12$.

3. Make r the subject of $S = \dfrac{a}{1 - r}$.

4. Simplify: $\left(1+\dfrac{1}{x}\right)\left(1-\dfrac{1}{x+1}\right)$.

5. Given $x+y=3$ and $xy=1$, find x^3+y^3.

6. Simplify: $\dfrac{(a+b)^2-(a-b)^2}{4ab}$.

7. Factorise: x^3+3x^2-x-3.

8. Solve: $\dfrac{x-1}{x+2}=\dfrac{x+3}{x+8}$.

9. Simplify: $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}+\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$.

10. The cubic $x^3-6x^2+11x-6=0$ has roots α,β,γ. Without solving it, find $\alpha^2+\beta^2+\gamma^2$ and $\alpha\beta\gamma$.

Section C Substitution & Structure

(≤90 s each)

Find structure before computing. Substitution is the key, not expansion.

1. Solve: $x^4-13x^2+36=0$.

2. Solve: $(x+1)(x+2)(x+3)(x+4)=24$.

3. Simplify: $\sqrt{6+2\sqrt{5}}$.

4. Given $a+b+c=1$ and $a^2+b^2+c^2=1$, find the value of $ab+bc+ca$ and hence evaluate $a^3+b^3+c^3-3abc$.

5. Solve: $(x-1)(x-3)(x-5)(x-7)+15=0$.

6. If $x=2+\sqrt{3}$, find $x^2+\dfrac{1}{x^2}$ without expanding x^2 directly.

7. Given $\dfrac{a}{b}+\dfrac{b}{a}=3$, find $\dfrac{a^2}{b^2}+\dfrac{b^2}{a^2}$.

8. If $x+y+z=0$, simplify $\dfrac{(x^2+y^2+z^2)^2}{x^4+y^4+z^4}$.

Section D Challenge

(TMUA / SMC difficulty)

Each has a solution under five lines. Find the insight, not the computation.

1. Evaluate:

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots+\frac{1}{8\cdot9\cdot10}.$$

2. The roots of $x^2 + px + q = 0$ are r and s. Find, in terms of p and q, the monic quadratic whose roots are $r^2 + s$ and $s^2 + r$.

3. The equation $\dfrac{x}{y+z} = \dfrac{y}{x+z} = \dfrac{z}{x+y} = k$ holds for real x, y, z with $x + y + z \neq 0$. Find all possible values of k.

4. Prove the **Brahmagupta–Fibonacci identity**:

$$(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2.$$

Hence express 85×65 as a sum of two perfect squares in two different ways.

5. Find all integers $n \geq 1$ for which $n^2 + n + 1$ divides $n^3 - 1$.

"An elegant solution is not luck — it is the reward for seeing the structure that was always there."

Found a mistake or want to contribute a sheet?
Visit the open-source project at github.com/The-CerealDev/speed-maths